STP 598

Homework 3, Fall 2024

This assignment uses strength data from HW2. The data set for this assignment is *unbalanced* over treatment groups. Participants in the study were followed for seven weeks, with a measurement of strength taken each week. The measurements were equally spaced over time. The study had three treatment groups: a group with weights increasing over time (WI), a group with repetitions increasing over time (RI), and a control group (CONT). Participants were randomly allocated to treatment groups, and subject are considered to be a sample from a larger population. The data are contained in the file strength.dat on Canvas. The data are in wide format, where each column represents successive measurements over the seven weeks of the study. The purpose of the study was to determine if either WI or RI produced larger increase in strength over time. You may use SAS, R, python, or Matlab to obtain answers to the questions below.

1. Consider the AR(1) and toeplitz covariance structures. Using symbols, write down the structure for these matrices for the first four measurements (i.e., the first four weeks). What are the elements of the ***α*** vector for each of these structures? The ***α*** vector contains the parameters for the covariance matrix.

AR(1): -1 < ρ <1

A math equations with numbers and symbols

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α = (σε2, ρ)

TOEP:

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α = (σε2, σ1, σ2, σ3,)

1. For the linear model ***Y*** *i* = ***X*** *i****β*** + ***ε****i* use the data in univariate format or long format and a **saturated** factor effects model for the mean response and fit the following models for the Σ**Y** covariance structure:

(a) unstructured, (b) compound symmetry, (c) AR(1), and (d) toeplitz. (The toeplitz model is available in R in the package glmmTMB. Use GLS for the other models.) Assume homogeneity of covariance matrix across treatment groups. Use either ML for estimation of all models or REML for estimation of all models. The default in GLS is REML, but ML is available as an option, method = “ML”. The default in gmmTMB is ML, but REML is available as an option, REML=TRUE. Using AIC, BIC and likelihood ratio difference tests, which covariance model is most appropriate for the data? Why? Do all information criteria and the likelihood ratio tests agree on the choice for the covariance model?

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- AR(1) Model 3 has the lowest AIC (1312.804) and BIC (1403.306), indicating it provides the best fit among the tested models when considering both criteria. Yet, Toeplitz Model 4 has the highest log-likelihood (-624.3451), meaning it fits the data better than the other models, even though it is penalized more by AIC and BIC for being more complex.

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Description automatically generated \* From the log-likelihood ratio test, AR(1) model is sufficient, and there is no significant improvement in fit with the more complex Toeplitz model (P>0.05).

All three —AIC, BIC, and the likelihood ratio test—are in agreement: AR(1) Model 3 is the most appropriate model for the data. The additional complexity of the Toeplitz model is not justified by a significant improvement in fit.

1. For the linear model ***Y*** *i* = ***X*** *i****β*** + ***ε****i*, using your selected covariance model, enter week as a factor effect variable.
   1. Fit a model which includes effects of week, treatment, and their interaction (weekf\*treatment). Use GLS or glmmTMB, depending on the covariance structure. weekf is week as a factor. Is treatment\*weekf significant? At which weeks is the difference among the treatment groups significant?

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Description automatically generatedIs treatment\*weekf significant?

No. P-value for the interaction is 0.03 > 0.05. Thus, the interaction between group and weekf is not statistically significant, implying that the group (treatment) effect on strength does not significantly differ across the weeks.

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At which weeks is the difference among the treatment groups significant?

The significant differences occurred mostly in the later weeks: At week 5, 6, and 7, both the RI and WI group show significant differences from the control group with p<0.05.

Significant differences between RI and the control group occur at Weeks 2, 5, 6, and 7. Significant differences between WI and the control group occur at Weeks 4, 5, 6, and 7.

* 1. Use the nested model without grand mean (-1 + weekf:treatment). Use orthogonal polynomials contrasts to test linear and quadratic trends in the WI treatment group and the RI group. Write down estimates of the trends and test statistics. Which trends are significant?

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The linear trend is significant for both the RI and WI groups, indicating that strength increases over time for both treatment groups.

The quadratic trend is significant only for the RI group, suggesting that strength improvement in the RI group is non-linear, while the WI group shows no significant quadratic trend.

* 1. Use a contrast with orthogonal polynomial coefficients to test the null hypothesis that the quadratic trend for WI is equal to the quadratic trend for RI. This is a contrast of contrasts. Write down estimate of contrast and test statistic. Is there a significant difference? Why or why not?

H0: beta\_quadratic\_WI = beta\_quadatic RI

H1: beta\_quadratic\_WI ≠ beta\_quadatic RI

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With p=0.3711226 > 0.05, there is no significant difference between the quadratic trends for the WI and RI groups. Thus, we **fail to reject** the null hypothesis. The data does not provide enough evidence to conclude that the rate of non-linear (quadratic) change in strength over time differs between the two groups.

1. For the linear model ***Y*** *i* = ***X*** *i****β*** + ***ε****i*. using week as a **continuous** variable.
   1. Estimate a model with linear, quadratic and cubic trends over time and interaction of polynomial trends by treatment group. Test week\*treatment, weeksq\*treatment and weekcubed\*treatment. Which polynomial trends differ by treatment group? Use the model for Σ**Y** selected above.

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Description automatically generatedWhile both the linear and quadratic trends are significant over time, only the **linear trend** shows significant differences between treatment groups. The quadratic and cubic trends do not significantly differ between the groups.

* 1. Use symbols and write down the elements of the ***β*** vector. Define the elements of the vector.

Yh(l)j = β0 + β1 (thj) + β2 (t2hj) + β3 (t3hj) + β4 (X1hj) + β5 (X2hj) + β6 (thj) (X1hj) + β7 (thj) (X2hj) + β8 (t2hj) (X1hj) + β9 (t2hj) (X2hj) + β10 (t3hj) (X1hj) + β11 (t3hj) (X2hj) + εnj

Yh(l)j = β0 + β1(week) + β2(week^2) + β3(week^3) + β4(group RI) + β5(group WI) + β6 (week \* RI) + β7 (week \* WI) + β8 (week^2 \* RI) + β9 (week^2 \* WI) + β10 (week^3 \* RI) + β11 (week^3 \* WI) + εhj

Elements of the β vector:

β’ = (β0, β1, β2, β3, β4, β5, β6, β7, β8, β9, β10, β11)

* ​h: individual h
* l: treatment group l (RI, WI)
* j: time (week 1…7)
* thj​: Time (week) for individual h in treatment group l.
* (X1hj) ​ and (X2hj) ​: Dummy variables for treatment groups. Thus, 1 if individual h is in the treatment group and zero otherwise.
* β0: intercept (baseline strength for CONT troup)
* β1: (thj):​ **linear time (week) effect** for the CONT group.
* β2: (t2hj):​ **quadratic time effect** for the CONT group
* β3: (t3hj):​ **cubic time effect** for the CONT group
* β4: (X1hj): **effect of RI**: baseline shift for group RI relative to the CONT group
* β5: (X2hj): **effect of WI**: baseline shift for group WI relative to the CONT group
* β6: (thj) (X1hj): The interaction between **linear time** and treatment 1 (group RI)
* β7: (thj) (X2hj): The interaction between **linear time** and treatment 2 (group WI)
* β8: (t2hj) (X1hj): The interaction between **quadratic time** and treatment 1 (group RI)
* β9: (t2hj) (X2hj): The interaction between **quadratic time** and treatment 2 (group WI)
* β10: (t3hj) (X1hj): The interaction between **cubic time** and treatment 1 (group RI)
* β11: (t3hj) (X2hj): The interaction between **cubic time** and treatment 2 (group WI)
* εnj: residual error

To simplify:

**Yh CONT= β0 CONT + β1CONT (t) + β2 CONT (t2) + β3 CONT (t3) + ε CONT**

**Yh RI= β0 RI + β1 RI (t) + β2 RI (t2) + β3 RI (t3) + ε RI**

**Yh WI= β0 WI + β1 WI (t) + β2 WI (t2) + β3 WI (t3) + ε WI**

β0 :intercept (baseline strength for each group)

β1 : linear time (week) effect for each group

β2: quadratic time effect for each group

β3: cubic time effect for each group

t: time

**Elements of the β vector:**

**β’ = (β0, β1, β2, β3,)**

* 1. Write down the ***X*** *i* matrix for an individual in the CONT group.

Y.bar = Xi β’ + ε

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* 1. Now use the nested model (-1 + week:treatment). Include week, weeksq, and weekcubed. Which trends are significant for CONT? for RI? for WI? Center week to reduce collinearity.

Conclusion: The linear trend for WI was marginally significant before centering, but centering led to it becoming non-significant. Before centering, the quadratic trend for RI was not significant. However, after centering, it became significant.

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While the linear and quadratic trends over time differ significantly between the treatment groups, in this nested model:

* For CONT,there is no significant linear (week\_num), quadratic (week\_sq), nor cubic (week\_cub) trend.
* For RI, there is significant linear trend (p=0.0178) but not quadratic or cubic trend.
* For WI, there is marginary significant linear trend (p=0.0578) but not quadratic nor cubic trend.

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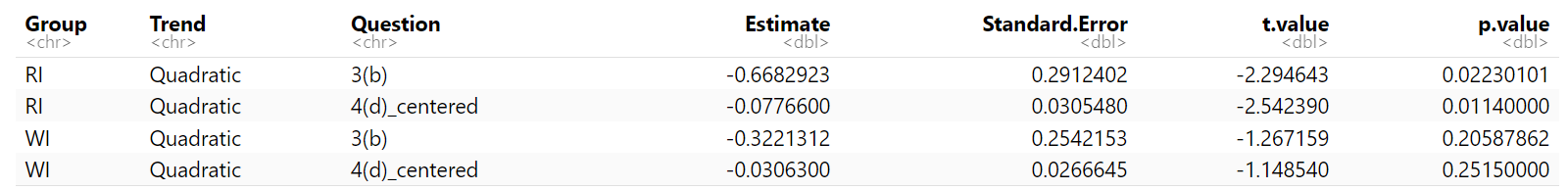
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In the centered model:

* For CONT, there is no significant linear (week\_c), quadratic (week\_c\_sq), nor cubic (week\_c\_cub) trend.
* For RI, there is no significant linear trendand, but a significant quadratic trend (p=0.0114), and no significant cubic trend.
* For WI, there is no to marginary significant linear (p=0.0790), no quadratic, and no cubic trend.
  1. Compare the test statistic using orthogonal polynomial contrasts for quadratic trend in 3(b) to the test statistics for quadratic trend in the previous question, 4(d). Are the test statistics the same? Why would the test statistics be different?



The test statistics differ because centering the week variable reduces the collinearity between the linear, quadratic, and cubic terms. Without centering, the higher-order polynomial terms may be more correlated with each other, inflating standard errors and reducing the precision of the estimates.

In this case, centering did not drastically change the overall conclusions about significance between before and after centering. Quadratic trend for RI is significant but not for WI group.

* 1. Write down the polynomial model for CONT. Write down the polynomial model for RI. Write down the polynomial model for WI. Include only significant terms in the models.

The models are from fit.ar1.poly.nested. To assess the significance of each trend, I conducted Type 1 sum of squares tests to compare the full model to reduced models (without linear, quadratic, and cubic terms). Linear terms are highly significant and quadratic terms are important for RI group.

* Polynomial model for CONT: No significant trend, flat

Y.hat\_CONT (t) = β0 CONT = 79.24492

* Polynomial model for RI: significant Linear and quadratic trend

Y.hat\_RI (t) = β0 RI + β1RI t + β1RI t 2 = 78.48772 + 1.44110 (t) -0.25619 (t^2)

* Polynomial model for WI: Marginally significant linear trend (p=0.0578)

Y.hat\_WI (t) = β0 RI + β1WI (t) = 80.19050 + 1.00617 (t)

How do the trends for RI and WI differ?

**RI** has a **quadratic trend**: strength for the RI group initially increases over time and then become flat due to the negative quadratic coefficient

**WI** shows a **linear trend**: strength consistently increases over time.

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R codes:

HW3\_1012

Jiseon Yang

2024-10-13

## Data

# Load necessary libraries  
library(nlme)  
library(glmmTMB)  
library(reshape2)  
  
# Load the data  
strength\_wide1 <- read.csv("C:/Users/jyang/OneDrive - Arizona State University/10 Classes\_OneDrive/2024 8F\_STP598\_Logitudinal/zz HW/Longitudinal\_HW3/strength.csv", header = FALSE)  
names(strength\_wide1) <- c("ID", "Group", "Week1", "Week2", "Week3", "Week4", "Week5", "Week6", "Week7")  
  
# Convert data from wide format to long format  
strength\_long1 <- melt(strength\_wide1, id.vars = c("ID", "Group"),  
 variable.name = "Week", value.name = "Strength")  
  
# Convert variables to factors  
strength\_long1$Weekf <- as.factor(strength\_long1$Week)  
strength\_long1$IDf <- as.factor(strength\_long1$ID)  
strength\_long1$Groupf <- as.factor(strength\_long1$Group)  
  
# Check levels to prevent errors  
if (length(levels(strength\_long1$Groupf)) < 2) stop("Group must have at least two levels.")  
if (length(levels(strength\_long1$Weekf)) < 2) stop("Week must have at least two levels.")  
  
# Convert `Week` to numeric after removing the "Week" prefix  
strength\_long1$Week\_num <- as.numeric(gsub("Week", "", strength\_long1$Weekf))  
  
# Remove rows with missing values  
strength\_long1 <- na.omit(strength\_long1)

## Question 2:

1. Unstructured Covariance Structure (Model 1)

fit.unstructured\_1 <- gls(Strength ~ Group + Weekf + Weekf\*Group,   
 data = strength\_long1,   
 correlation = corSymm(form = ~ 1 | IDf),   
 weights = varIdent(form = ~ 1 | Weekf),   
 method = "REML") # or "ML"  
summary(fit.unstructured\_1)

## Generalized least squares fit by REML  
## Model: Strength ~ Group + Weekf + Weekf \* Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1332.896 1525.706 -617.4479  
##   
## Correlation Structure: General  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Correlation:   
## 1 2 3 4 5 6   
## 2 0.960   
## 3 0.925 0.940   
## 4 0.872 0.877 0.956   
## 5 0.842 0.860 0.937 0.960   
## 6 0.809 0.827 0.898 0.909 0.951   
## 7 0.797 0.792 0.876 0.887 0.917 0.953  
## Variance function:  
## Structure: Different standard deviations per stratum  
## Formula: ~1 | Weekf   
## Parameter estimates:  
## Week1 Week2 Week3 Week4 Week5 Week6 Week7   
## 1.000000 1.038705 1.104340 1.071325 1.173689 1.157116 1.203158   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## (Intercept) 79.75000 0.6625852 120.36187 0.0000  
## GroupRI -0.06250 0.9938779 -0.06288 0.9499  
## GroupWI 1.29762 0.9258146 1.40160 0.1619  
## WeekfWeek2 0.20000 0.1922152 1.04050 0.2988  
## WeekfWeek3 0.25000 0.2790045 0.89604 0.3708  
## WeekfWeek4 0.30000 0.3507594 0.85529 0.3929  
## WeekfWeek5 0.05000 0.4194662 0.11920 0.9052  
## WeekfWeek6 -0.15000 0.4525173 -0.33148 0.7405  
## WeekfWeek7 -0.15000 0.4825102 -0.31087 0.7561  
## GroupRI:WeekfWeek2 0.67500 0.2883228 2.34113 0.0197  
## GroupWI:WeekfWeek2 0.41905 0.2685777 1.56025 0.1195  
## GroupRI:WeekfWeek3 0.87500 0.4185067 2.09077 0.0372  
## GroupWI:WeekfWeek3 0.60714 0.3898464 1.55739 0.1202  
## GroupRI:WeekfWeek4 1.01250 0.5261391 1.92440 0.0551  
## GroupWI:WeekfWeek4 1.17619 0.4901078 2.39986 0.0169  
## GroupRI:WeekfWeek5 1.51250 0.6291993 2.40385 0.0167  
## GroupWI:WeekfWeek5 1.52143 0.5861102 2.59581 0.0098  
## GroupRI:WeekfWeek6 1.58750 0.6787759 2.33877 0.0199  
## GroupWI:WeekfWeek6 1.81667 0.6322917 2.87315 0.0043  
## GroupRI:WeekfWeek7 1.58750 0.7237652 2.19339 0.0289  
## GroupWI:WeekfWeek7 2.19762 0.6742000 3.25960 0.0012  
##   
## Correlation:   
## (Intr) GropRI GropWI WkfWk2 WkfWk3 WkfWk4 WkfWk5 WkfWk6  
## GroupRI -0.667   
## GroupWI -0.716 0.477   
## WeekfWeek2 -0.009 0.006 0.006   
## WeekfWeek3 0.050 -0.033 -0.036 0.485   
## WeekfWeek4 -0.125 0.083 0.090 0.292 0.788   
## WeekfWeek5 -0.018 0.012 0.013 0.339 0.770 0.851   
## WeekfWeek6 -0.093 0.062 0.067 0.307 0.659 0.712 0.850   
## WeekfWeek7 -0.057 0.038 0.041 0.158 0.599 0.652 0.753 0.869  
## GroupRI:WeekfWeek2 0.006 -0.009 -0.004 -0.667 -0.324 -0.194 -0.226 -0.204  
## GroupWI:WeekfWeek2 0.006 -0.004 -0.009 -0.716 -0.347 -0.209 -0.242 -0.219  
## GroupRI:WeekfWeek3 -0.033 0.050 0.024 -0.324 -0.667 -0.525 -0.513 -0.439  
## GroupWI:WeekfWeek3 -0.036 0.024 0.050 -0.347 -0.716 -0.564 -0.551 -0.472  
## GroupRI:WeekfWeek4 0.083 -0.125 -0.060 -0.194 -0.525 -0.667 -0.567 -0.475  
## GroupWI:WeekfWeek4 0.090 -0.060 -0.125 -0.209 -0.564 -0.716 -0.609 -0.510  
## GroupRI:WeekfWeek5 0.012 -0.018 -0.009 -0.226 -0.513 -0.567 -0.667 -0.567  
## GroupWI:WeekfWeek5 0.013 -0.009 -0.018 -0.242 -0.551 -0.609 -0.716 -0.608  
## GroupRI:WeekfWeek6 0.062 -0.093 -0.045 -0.204 -0.439 -0.475 -0.567 -0.667  
## GroupWI:WeekfWeek6 0.067 -0.045 -0.093 -0.219 -0.472 -0.510 -0.608 -0.716  
## GroupRI:WeekfWeek7 0.038 -0.057 -0.027 -0.106 -0.399 -0.435 -0.502 -0.579  
## GroupWI:WeekfWeek7 0.041 -0.027 -0.057 -0.113 -0.428 -0.467 -0.539 -0.622  
## WkfWk7 GRI:WW2 GWI:WW2 GRI:WW3 GWI:WW3 GRI:WW4 GWI:WW4  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2 -0.106   
## GroupWI:WeekfWeek2 -0.113 0.477   
## GroupRI:WeekfWeek3 -0.399 0.485 0.232   
## GroupWI:WeekfWeek3 -0.428 0.232 0.485 0.477   
## GroupRI:WeekfWeek4 -0.435 0.292 0.139 0.788 0.376   
## GroupWI:WeekfWeek4 -0.467 0.139 0.292 0.376 0.788 0.477   
## GroupRI:WeekfWeek5 -0.502 0.339 0.162 0.770 0.367 0.851 0.406   
## GroupWI:WeekfWeek5 -0.539 0.162 0.339 0.367 0.770 0.406 0.851   
## GroupRI:WeekfWeek6 -0.579 0.307 0.146 0.659 0.314 0.712 0.340   
## GroupWI:WeekfWeek6 -0.622 0.146 0.307 0.314 0.659 0.340 0.712   
## GroupRI:WeekfWeek7 -0.667 0.158 0.076 0.599 0.286 0.652 0.311   
## GroupWI:WeekfWeek7 -0.716 0.076 0.158 0.286 0.599 0.311 0.652   
## GRI:WW5 GWI:WW5 GRI:WW6 GWI:WW6 GRI:WW7  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2   
## GroupWI:WeekfWeek2   
## GroupRI:WeekfWeek3   
## GroupWI:WeekfWeek3   
## GroupRI:WeekfWeek4   
## GroupWI:WeekfWeek4   
## GroupRI:WeekfWeek5   
## GroupWI:WeekfWeek5 0.477   
## GroupRI:WeekfWeek6 0.850 0.405   
## GroupWI:WeekfWeek6 0.405 0.850 0.477   
## GroupRI:WeekfWeek7 0.753 0.359 0.869 0.414   
## GroupWI:WeekfWeek7 0.359 0.753 0.414 0.869 0.477   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.37840421 -0.63966179 -0.02671352 0.65888225 2.77942211   
##   
## Residual standard error: 2.963171   
## Degrees of freedom: 399 total; 378 residual

# Residual standard error: 2.963126   
# Degrees of freedom: 399 total; 378 residual  
  
fit.unstructured\_2 <- gls(Strength ~ -1 + Weekf:Group,   
 data = strength\_long1,   
 correlation = corSymm(form = ~ 1 | IDf),   
 weights = varIdent(form = ~ 1 | Weekf),   
 method = "REML") # or "ML"  
summary(fit.unstructured\_2)

## Generalized least squares fit by REML  
## Model: Strength ~ -1 + Weekf:Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1332.896 1525.706 -617.4479  
##   
## Correlation Structure: General  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Correlation:   
## 1 2 3 4 5 6   
## 2 0.960   
## 3 0.925 0.940   
## 4 0.872 0.877 0.956   
## 5 0.842 0.860 0.937 0.960   
## 6 0.809 0.827 0.898 0.909 0.951   
## 7 0.797 0.792 0.876 0.887 0.917 0.953  
## Variance function:  
## Structure: Different standard deviations per stratum  
## Formula: ~1 | Weekf   
## Parameter estimates:  
## Week1 Week2 Week3 Week4 Week5 Week6 Week7   
## 1.000000 1.038704 1.104340 1.071328 1.173691 1.157124 1.203163   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## WeekfWeek1:GroupCONT 79.75000 0.6625752 120.36370 0  
## WeekfWeek2:GroupCONT 79.95000 0.6882198 116.16928 0  
## WeekfWeek3:GroupCONT 80.00000 0.7317086 109.33315 0  
## WeekfWeek4:GroupCONT 80.05000 0.7098351 112.77267 0  
## WeekfWeek5:GroupCONT 79.80000 0.7776582 102.61577 0  
## WeekfWeek6:GroupCONT 79.60000 0.7666817 103.82405 0  
## WeekfWeek7:GroupCONT 79.60000 0.7971861 99.85122 0  
## WeekfWeek1:GroupRI 79.68750 0.7407816 107.57220 0  
## WeekfWeek2:GroupRI 80.56250 0.7694531 104.70099 0  
## WeekfWeek3:GroupRI 80.81250 0.8180750 98.78373 0  
## WeekfWeek4:GroupRI 81.00000 0.7936197 102.06399 0  
## WeekfWeek5:GroupRI 81.25000 0.8694484 93.45006 0  
## WeekfWeek6:GroupRI 81.12500 0.8571762 94.64215 0  
## WeekfWeek7:GroupRI 81.12500 0.8912811 91.02067 0  
## WeekfWeek1:GroupWI 81.04762 0.6466072 125.34291 0  
## WeekfWeek2:GroupWI 81.66667 0.6716337 121.59405 0  
## WeekfWeek3:GroupWI 81.90476 0.7140744 114.70059 0  
## WeekfWeek4:GroupWI 82.52381 0.6927281 119.12872 0  
## WeekfWeek5:GroupWI 82.61905 0.7589167 108.86444 0  
## WeekfWeek6:GroupWI 82.71429 0.7482047 110.55034 0  
## WeekfWeek7:GroupWI 83.09524 0.7779739 106.80980 0  
##   
## Correlation:   
## WW1:GC WW2:GC WW3:GC WW4:GC WW5:GC WW6:GC WW7:GC WW1:GR  
## WeekfWeek2:GroupCONT 0.960   
## WeekfWeek3:GroupCONT 0.925 0.940   
## WeekfWeek4:GroupCONT 0.872 0.877 0.956   
## WeekfWeek5:GroupCONT 0.842 0.860 0.937 0.960   
## WeekfWeek6:GroupCONT 0.809 0.827 0.898 0.909 0.951   
## WeekfWeek7:GroupCONT 0.797 0.792 0.876 0.887 0.917 0.953   
## WeekfWeek1:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.960   
## WeekfWeek3:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.925   
## WeekfWeek4:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.872   
## WeekfWeek5:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.842   
## WeekfWeek6:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.809   
## WeekfWeek7:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.797   
## WeekfWeek1:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek3:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek4:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek5:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek6:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek7:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WW2:GR WW3:GR WW4:GR WW5:GR WW6:GR WW7:GR WW1:GW WW2:GW  
## WeekfWeek2:GroupCONT   
## WeekfWeek3:GroupCONT   
## WeekfWeek4:GroupCONT   
## WeekfWeek5:GroupCONT   
## WeekfWeek6:GroupCONT   
## WeekfWeek7:GroupCONT   
## WeekfWeek1:GroupRI   
## WeekfWeek2:GroupRI   
## WeekfWeek3:GroupRI 0.940   
## WeekfWeek4:GroupRI 0.877 0.956   
## WeekfWeek5:GroupRI 0.860 0.937 0.960   
## WeekfWeek6:GroupRI 0.827 0.898 0.909 0.951   
## WeekfWeek7:GroupRI 0.792 0.876 0.887 0.917 0.953   
## WeekfWeek1:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.960   
## WeekfWeek3:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.925 0.940   
## WeekfWeek4:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.872 0.877   
## WeekfWeek5:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.842 0.860   
## WeekfWeek6:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.809 0.827   
## WeekfWeek7:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.797 0.792   
## WW3:GW WW4:GW WW5:GW WW6:GW  
## WeekfWeek2:GroupCONT   
## WeekfWeek3:GroupCONT   
## WeekfWeek4:GroupCONT   
## WeekfWeek5:GroupCONT   
## WeekfWeek6:GroupCONT   
## WeekfWeek7:GroupCONT   
## WeekfWeek1:GroupRI   
## WeekfWeek2:GroupRI   
## WeekfWeek3:GroupRI   
## WeekfWeek4:GroupRI   
## WeekfWeek5:GroupRI   
## WeekfWeek6:GroupRI   
## WeekfWeek7:GroupRI   
## WeekfWeek1:GroupWI   
## WeekfWeek2:GroupWI   
## WeekfWeek3:GroupWI   
## WeekfWeek4:GroupWI 0.956   
## WeekfWeek5:GroupWI 0.937 0.960   
## WeekfWeek6:GroupWI 0.898 0.909 0.951   
## WeekfWeek7:GroupWI 0.876 0.887 0.917 0.953   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.37844038 -0.63967106 -0.02671382 0.65889227 2.77946327   
##   
## Residual standard error: 2.963126   
## Degrees of freedom: 399 total; 378 residual

1. Compound Symmetry (CS) Structure (Model 2)

fit.cs\_1 <- gls(Strength ~ Group + Weekf + Weekf\*Group,   
 data = strength\_long1,   
 correlation = corCompSymm(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.cs\_1)

## Generalized least squares fit by REML  
## Model: Strength ~ Group + Weekf + Weekf \* Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1466.82 1557.323 -710.4101  
##   
## Correlation Structure: Compound symmetry  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Rho   
## 0.8891805   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## (Intercept) 79.75000 0.7348538 108.52498 0.0000  
## GroupRI -0.06250 1.1022808 -0.05670 0.9548  
## GroupWI 1.29762 1.0267938 1.26376 0.2071  
## WeekfWeek2 0.20000 0.3459585 0.57810 0.5635  
## WeekfWeek3 0.25000 0.3459585 0.72263 0.4704  
## WeekfWeek4 0.30000 0.3459585 0.86716 0.3864  
## WeekfWeek5 0.05000 0.3459585 0.14453 0.8852  
## WeekfWeek6 -0.15000 0.3459585 -0.43358 0.6648  
## WeekfWeek7 -0.15000 0.3459585 -0.43358 0.6648  
## GroupRI:WeekfWeek2 0.67500 0.5189377 1.30073 0.1941  
## GroupWI:WeekfWeek2 0.41905 0.4833996 0.86688 0.3866  
## GroupRI:WeekfWeek3 0.87500 0.5189377 1.68614 0.0926  
## GroupWI:WeekfWeek3 0.60714 0.4833996 1.25599 0.2099  
## GroupRI:WeekfWeek4 1.01250 0.5189377 1.95110 0.0518  
## GroupWI:WeekfWeek4 1.17619 0.4833996 2.43316 0.0154  
## GroupRI:WeekfWeek5 1.51250 0.5189377 2.91461 0.0038  
## GroupWI:WeekfWeek5 1.52143 0.4833996 3.14735 0.0018  
## GroupRI:WeekfWeek6 1.58750 0.5189377 3.05913 0.0024  
## GroupWI:WeekfWeek6 1.81667 0.4833996 3.75811 0.0002  
## GroupRI:WeekfWeek7 1.58750 0.5189377 3.05913 0.0024  
## GroupWI:WeekfWeek7 2.19762 0.4833996 4.54618 0.0000  
##   
## Correlation:   
## (Intr) GropRI GropWI WkfWk2 WkfWk3 WkfWk4 WkfWk5 WkfWk6  
## GroupRI -0.667   
## GroupWI -0.716 0.477   
## WeekfWeek2 -0.235 0.157 0.168   
## WeekfWeek3 -0.235 0.157 0.168 0.500   
## WeekfWeek4 -0.235 0.157 0.168 0.500 0.500   
## WeekfWeek5 -0.235 0.157 0.168 0.500 0.500 0.500   
## WeekfWeek6 -0.235 0.157 0.168 0.500 0.500 0.500 0.500   
## WeekfWeek7 -0.235 0.157 0.168 0.500 0.500 0.500 0.500 0.500  
## GroupRI:WeekfWeek2 0.157 -0.235 -0.112 -0.667 -0.333 -0.333 -0.333 -0.333  
## GroupWI:WeekfWeek2 0.168 -0.112 -0.235 -0.716 -0.358 -0.358 -0.358 -0.358  
## GroupRI:WeekfWeek3 0.157 -0.235 -0.112 -0.333 -0.667 -0.333 -0.333 -0.333  
## GroupWI:WeekfWeek3 0.168 -0.112 -0.235 -0.358 -0.716 -0.358 -0.358 -0.358  
## GroupRI:WeekfWeek4 0.157 -0.235 -0.112 -0.333 -0.333 -0.667 -0.333 -0.333  
## GroupWI:WeekfWeek4 0.168 -0.112 -0.235 -0.358 -0.358 -0.716 -0.358 -0.358  
## GroupRI:WeekfWeek5 0.157 -0.235 -0.112 -0.333 -0.333 -0.333 -0.667 -0.333  
## GroupWI:WeekfWeek5 0.168 -0.112 -0.235 -0.358 -0.358 -0.358 -0.716 -0.358  
## GroupRI:WeekfWeek6 0.157 -0.235 -0.112 -0.333 -0.333 -0.333 -0.333 -0.667  
## GroupWI:WeekfWeek6 0.168 -0.112 -0.235 -0.358 -0.358 -0.358 -0.358 -0.716  
## GroupRI:WeekfWeek7 0.157 -0.235 -0.112 -0.333 -0.333 -0.333 -0.333 -0.333  
## GroupWI:WeekfWeek7 0.168 -0.112 -0.235 -0.358 -0.358 -0.358 -0.358 -0.358  
## WkfWk7 GRI:WW2 GWI:WW2 GRI:WW3 GWI:WW3 GRI:WW4 GWI:WW4  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2 -0.333   
## GroupWI:WeekfWeek2 -0.358 0.477   
## GroupRI:WeekfWeek3 -0.333 0.500 0.239   
## GroupWI:WeekfWeek3 -0.358 0.239 0.500 0.477   
## GroupRI:WeekfWeek4 -0.333 0.500 0.239 0.500 0.239   
## GroupWI:WeekfWeek4 -0.358 0.239 0.500 0.239 0.500 0.477   
## GroupRI:WeekfWeek5 -0.333 0.500 0.239 0.500 0.239 0.500 0.239   
## GroupWI:WeekfWeek5 -0.358 0.239 0.500 0.239 0.500 0.239 0.500   
## GroupRI:WeekfWeek6 -0.333 0.500 0.239 0.500 0.239 0.500 0.239   
## GroupWI:WeekfWeek6 -0.358 0.239 0.500 0.239 0.500 0.239 0.500   
## GroupRI:WeekfWeek7 -0.667 0.500 0.239 0.500 0.239 0.500 0.239   
## GroupWI:WeekfWeek7 -0.716 0.239 0.500 0.239 0.500 0.239 0.500   
## GRI:WW5 GWI:WW5 GRI:WW6 GWI:WW6 GRI:WW7  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2   
## GroupWI:WeekfWeek2   
## GroupRI:WeekfWeek3   
## GroupWI:WeekfWeek3   
## GroupRI:WeekfWeek4   
## GroupWI:WeekfWeek4   
## GroupRI:WeekfWeek5   
## GroupWI:WeekfWeek5 0.477   
## GroupRI:WeekfWeek6 0.500 0.239   
## GroupWI:WeekfWeek6 0.239 0.500 0.477   
## GroupRI:WeekfWeek7 0.500 0.239 0.500 0.239   
## GroupWI:WeekfWeek7 0.239 0.500 0.239 0.500 0.477   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.31838053 -0.62306477 -0.02897976 0.63755465 2.82552627   
##   
## Residual standard error: 3.286366   
## Degrees of freedom: 399 total; 378 residual

# Rho = 0.8891805  
# Residual standard error: 3.286366   
# Degrees of freedom: 399 total; 378 residual  
  
fit.cs\_2 <- gls(Strength ~ -1 + Weekf:Group,   
 data = strength\_long1,   
 correlation = corCompSymm(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.cs\_2)

## Generalized least squares fit by REML  
## Model: Strength ~ -1 + Weekf:Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1466.82 1557.323 -710.4101  
##   
## Correlation Structure: Compound symmetry  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Rho   
## 0.8891805   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## WeekfWeek1:GroupCONT 79.75000 0.7348538 108.52498 0  
## WeekfWeek2:GroupCONT 79.95000 0.7348538 108.79715 0  
## WeekfWeek3:GroupCONT 80.00000 0.7348538 108.86519 0  
## WeekfWeek4:GroupCONT 80.05000 0.7348538 108.93323 0  
## WeekfWeek5:GroupCONT 79.80000 0.7348538 108.59302 0  
## WeekfWeek6:GroupCONT 79.60000 0.7348538 108.32086 0  
## WeekfWeek7:GroupCONT 79.60000 0.7348538 108.32086 0  
## WeekfWeek1:GroupRI 79.68750 0.8215916 96.99162 0  
## WeekfWeek2:GroupRI 80.56250 0.8215916 98.05663 0  
## WeekfWeek3:GroupRI 80.81250 0.8215916 98.36092 0  
## WeekfWeek4:GroupRI 81.00000 0.8215916 98.58913 0  
## WeekfWeek5:GroupRI 81.25000 0.8215916 98.89342 0  
## WeekfWeek6:GroupRI 81.12500 0.8215916 98.74128 0  
## WeekfWeek7:GroupRI 81.12500 0.8215916 98.74128 0  
## WeekfWeek1:GroupWI 81.04762 0.7171439 113.01444 0  
## WeekfWeek2:GroupWI 81.66667 0.7171439 113.87765 0  
## WeekfWeek3:GroupWI 81.90476 0.7171439 114.20966 0  
## WeekfWeek4:GroupWI 82.52381 0.7171439 115.07287 0  
## WeekfWeek5:GroupWI 82.61905 0.7171439 115.20567 0  
## WeekfWeek6:GroupWI 82.71429 0.7171439 115.33847 0  
## WeekfWeek7:GroupWI 83.09524 0.7171439 115.86968 0  
##   
## Correlation:   
## WW1:GC WW2:GC WW3:GC WW4:GC WW5:GC WW6:GC WW7:GC WW1:GR  
## WeekfWeek2:GroupCONT 0.889   
## WeekfWeek3:GroupCONT 0.889 0.889   
## WeekfWeek4:GroupCONT 0.889 0.889 0.889   
## WeekfWeek5:GroupCONT 0.889 0.889 0.889 0.889   
## WeekfWeek6:GroupCONT 0.889 0.889 0.889 0.889 0.889   
## WeekfWeek7:GroupCONT 0.889 0.889 0.889 0.889 0.889 0.889   
## WeekfWeek1:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.889   
## WeekfWeek3:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.889   
## WeekfWeek4:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.889   
## WeekfWeek5:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.889   
## WeekfWeek6:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.889   
## WeekfWeek7:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.889   
## WeekfWeek1:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek3:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek4:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek5:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek6:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek7:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WW2:GR WW3:GR WW4:GR WW5:GR WW6:GR WW7:GR WW1:GW WW2:GW  
## WeekfWeek2:GroupCONT   
## WeekfWeek3:GroupCONT   
## WeekfWeek4:GroupCONT   
## WeekfWeek5:GroupCONT   
## WeekfWeek6:GroupCONT   
## WeekfWeek7:GroupCONT   
## WeekfWeek1:GroupRI   
## WeekfWeek2:GroupRI   
## WeekfWeek3:GroupRI 0.889   
## WeekfWeek4:GroupRI 0.889 0.889   
## WeekfWeek5:GroupRI 0.889 0.889 0.889   
## WeekfWeek6:GroupRI 0.889 0.889 0.889 0.889   
## WeekfWeek7:GroupRI 0.889 0.889 0.889 0.889 0.889   
## WeekfWeek1:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.889   
## WeekfWeek3:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.889 0.889   
## WeekfWeek4:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.889 0.889   
## WeekfWeek5:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.889 0.889   
## WeekfWeek6:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.889 0.889   
## WeekfWeek7:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.889 0.889   
## WW3:GW WW4:GW WW5:GW WW6:GW  
## WeekfWeek2:GroupCONT   
## WeekfWeek3:GroupCONT   
## WeekfWeek4:GroupCONT   
## WeekfWeek5:GroupCONT   
## WeekfWeek6:GroupCONT   
## WeekfWeek7:GroupCONT   
## WeekfWeek1:GroupRI   
## WeekfWeek2:GroupRI   
## WeekfWeek3:GroupRI   
## WeekfWeek4:GroupRI   
## WeekfWeek5:GroupRI   
## WeekfWeek6:GroupRI   
## WeekfWeek7:GroupRI   
## WeekfWeek1:GroupWI   
## WeekfWeek2:GroupWI   
## WeekfWeek3:GroupWI   
## WeekfWeek4:GroupWI 0.889   
## WeekfWeek5:GroupWI 0.889 0.889   
## WeekfWeek6:GroupWI 0.889 0.889 0.889   
## WeekfWeek7:GroupWI 0.889 0.889 0.889 0.889   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.31838053 -0.62306477 -0.02897976 0.63755465 2.82552627   
##   
## Residual standard error: 3.286366   
## Degrees of freedom: 399 total; 378 residual

1. AR(1) Structure (Model 3)

fit.ar1\_1 <- gls(Strength ~ Group + Weekf + Weekf\*Group,   
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.ar1\_1)

## Generalized least squares fit by REML  
## Model: Strength ~ Group + Weekf + Weekf \* Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1312.804 1403.306 -633.4018  
##   
## Correlation Structure: AR(1)  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Phi   
## 0.9517769   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## (Intercept) 79.75000 0.7334844 108.72760 0.0000  
## GroupRI -0.06250 1.1002267 -0.05681 0.9547  
## GroupWI 1.29762 1.0248804 1.26612 0.2063  
## WeekfWeek2 0.20000 0.2277894 0.87800 0.3805  
## WeekfWeek3 0.25000 0.3182354 0.78558 0.4326  
## WeekfWeek4 0.30000 0.3850689 0.77908 0.4364  
## WeekfWeek5 0.05000 0.4393354 0.11381 0.9095  
## WeekfWeek6 -0.15000 0.4853821 -0.30903 0.7575  
## WeekfWeek7 -0.15000 0.5254737 -0.28546 0.7755  
## GroupRI:WeekfWeek2 0.67500 0.3416840 1.97551 0.0489  
## GroupWI:WeekfWeek2 0.41905 0.3182847 1.31658 0.1888  
## GroupRI:WeekfWeek3 0.87500 0.4773531 1.83302 0.0676  
## GroupWI:WeekfWeek3 0.60714 0.4446628 1.36540 0.1729  
## GroupRI:WeekfWeek4 1.01250 0.5776033 1.75293 0.0804  
## GroupWI:WeekfWeek4 1.17619 0.5380476 2.18603 0.0294  
## GroupRI:WeekfWeek5 1.51250 0.6590031 2.29513 0.0223  
## GroupWI:WeekfWeek5 1.52143 0.6138730 2.47841 0.0136  
## GroupRI:WeekfWeek6 1.58750 0.7280732 2.18041 0.0298  
## GroupWI:WeekfWeek6 1.81667 0.6782129 2.67861 0.0077  
## GroupRI:WeekfWeek7 1.58750 0.7882106 2.01406 0.0447  
## GroupWI:WeekfWeek7 2.19762 0.7342320 2.99309 0.0029  
##   
## Correlation:   
## (Intr) GropRI GropWI WkfWk2 WkfWk3 WkfWk4 WkfWk5 WkfWk6  
## GroupRI -0.667   
## GroupWI -0.716 0.477   
## WeekfWeek2 -0.155 0.104 0.111   
## WeekfWeek3 -0.217 0.145 0.155 0.699   
## WeekfWeek4 -0.262 0.175 0.188 0.564 0.807   
## WeekfWeek5 -0.299 0.200 0.214 0.483 0.690 0.855   
## WeekfWeek6 -0.331 0.221 0.237 0.427 0.610 0.756 0.883   
## WeekfWeek7 -0.358 0.239 0.256 0.386 0.551 0.682 0.797 0.901  
## GroupRI:WeekfWeek2 0.104 -0.155 -0.074 -0.667 -0.466 -0.376 -0.322 -0.285  
## GroupWI:WeekfWeek2 0.111 -0.074 -0.155 -0.716 -0.500 -0.403 -0.346 -0.306  
## GroupRI:WeekfWeek3 0.145 -0.217 -0.104 -0.466 -0.667 -0.538 -0.460 -0.407  
## GroupWI:WeekfWeek3 0.155 -0.104 -0.217 -0.500 -0.716 -0.577 -0.494 -0.437  
## GroupRI:WeekfWeek4 0.175 -0.262 -0.125 -0.376 -0.538 -0.667 -0.570 -0.504  
## GroupWI:WeekfWeek4 0.188 -0.125 -0.262 -0.403 -0.577 -0.716 -0.612 -0.541  
## GroupRI:WeekfWeek5 0.200 -0.299 -0.143 -0.322 -0.460 -0.570 -0.667 -0.589  
## GroupWI:WeekfWeek5 0.214 -0.143 -0.299 -0.346 -0.494 -0.612 -0.716 -0.632  
## GroupRI:WeekfWeek6 0.221 -0.331 -0.158 -0.285 -0.407 -0.504 -0.589 -0.667  
## GroupWI:WeekfWeek6 0.237 -0.158 -0.331 -0.306 -0.437 -0.541 -0.632 -0.716  
## GroupRI:WeekfWeek7 0.239 -0.358 -0.171 -0.257 -0.368 -0.455 -0.531 -0.601  
## GroupWI:WeekfWeek7 0.256 -0.171 -0.358 -0.276 -0.395 -0.488 -0.570 -0.645  
## WkfWk7 GRI:WW2 GWI:WW2 GRI:WW3 GWI:WW3 GRI:WW4 GWI:WW4  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2 -0.257   
## GroupWI:WeekfWeek2 -0.276 0.477   
## GroupRI:WeekfWeek3 -0.368 0.699 0.333   
## GroupWI:WeekfWeek3 -0.395 0.333 0.699 0.477   
## GroupRI:WeekfWeek4 -0.455 0.564 0.269 0.807 0.385   
## GroupWI:WeekfWeek4 -0.488 0.269 0.564 0.385 0.807 0.477   
## GroupRI:WeekfWeek5 -0.531 0.483 0.230 0.690 0.329 0.855 0.408   
## GroupWI:WeekfWeek5 -0.570 0.230 0.483 0.329 0.690 0.408 0.855   
## GroupRI:WeekfWeek6 -0.601 0.427 0.204 0.610 0.291 0.756 0.361   
## GroupWI:WeekfWeek6 -0.645 0.204 0.427 0.291 0.610 0.361 0.756   
## GroupRI:WeekfWeek7 -0.667 0.386 0.184 0.551 0.263 0.682 0.326   
## GroupWI:WeekfWeek7 -0.716 0.184 0.386 0.263 0.551 0.326 0.682   
## GRI:WW5 GWI:WW5 GRI:WW6 GWI:WW6 GRI:WW7  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2   
## GroupWI:WeekfWeek2   
## GroupRI:WeekfWeek3   
## GroupWI:WeekfWeek3   
## GroupRI:WeekfWeek4   
## GroupWI:WeekfWeek4   
## GroupRI:WeekfWeek5   
## GroupWI:WeekfWeek5 0.477   
## GroupRI:WeekfWeek6 0.883 0.421   
## GroupWI:WeekfWeek6 0.421 0.883 0.477   
## GroupRI:WeekfWeek7 0.797 0.380 0.901 0.430   
## GroupWI:WeekfWeek7 0.380 0.797 0.430 0.901 0.477   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.32270892 -0.62422802 -0.02903386 0.63874495 2.83080150   
##   
## Residual standard error: 3.280242   
## Degrees of freedom: 399 total; 378 residual

# Phi = 0.9517769  
# Residual standard error: 3.280242   
# Degrees of freedom: 399 total; 378 residual  
  
fit.ar1\_2 <- gls(Strength ~ -1 + Weekf:Group,   
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.ar1\_2)

## Generalized least squares fit by REML  
## Model: Strength ~ -1 + Weekf:Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1312.804 1403.306 -633.4018  
##   
## Correlation Structure: AR(1)  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Phi   
## 0.9517769   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## WeekfWeek1:GroupCONT 79.75000 0.7334844 108.72760 0  
## WeekfWeek2:GroupCONT 79.95000 0.7334844 109.00027 0  
## WeekfWeek3:GroupCONT 80.00000 0.7334844 109.06844 0  
## WeekfWeek4:GroupCONT 80.05000 0.7334844 109.13660 0  
## WeekfWeek5:GroupCONT 79.80000 0.7334844 108.79576 0  
## WeekfWeek6:GroupCONT 79.60000 0.7334844 108.52309 0  
## WeekfWeek7:GroupCONT 79.60000 0.7334844 108.52309 0  
## WeekfWeek1:GroupRI 79.68750 0.8200605 97.17271 0  
## WeekfWeek2:GroupRI 80.56250 0.8200605 98.23970 0  
## WeekfWeek3:GroupRI 80.81250 0.8200605 98.54456 0  
## WeekfWeek4:GroupRI 81.00000 0.8200605 98.77320 0  
## WeekfWeek5:GroupRI 81.25000 0.8200605 99.07805 0  
## WeekfWeek6:GroupRI 81.12500 0.8200605 98.92562 0  
## WeekfWeek7:GroupRI 81.12500 0.8200605 98.92562 0  
## WeekfWeek1:GroupWI 81.04762 0.7158075 113.22544 0  
## WeekfWeek2:GroupWI 81.66667 0.7158075 114.09026 0  
## WeekfWeek3:GroupWI 81.90476 0.7158075 114.42289 0  
## WeekfWeek4:GroupWI 82.52381 0.7158075 115.28771 0  
## WeekfWeek5:GroupWI 82.61905 0.7158075 115.42076 0  
## WeekfWeek6:GroupWI 82.71429 0.7158075 115.55381 0  
## WeekfWeek7:GroupWI 83.09524 0.7158075 116.08601 0  
##   
## Correlation:   
## WW1:GC WW2:GC WW3:GC WW4:GC WW5:GC WW6:GC WW7:GC WW1:GR  
## WeekfWeek2:GroupCONT 0.952   
## WeekfWeek3:GroupCONT 0.906 0.952   
## WeekfWeek4:GroupCONT 0.862 0.906 0.952   
## WeekfWeek5:GroupCONT 0.821 0.862 0.906 0.952   
## WeekfWeek6:GroupCONT 0.781 0.821 0.862 0.906 0.952   
## WeekfWeek7:GroupCONT 0.743 0.781 0.821 0.862 0.906 0.952   
## WeekfWeek1:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.952   
## WeekfWeek3:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.906   
## WeekfWeek4:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.862   
## WeekfWeek5:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.821   
## WeekfWeek6:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.781   
## WeekfWeek7:GroupRI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.743   
## WeekfWeek1:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek3:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek4:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek5:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek6:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek7:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000   
## WW2:GR WW3:GR WW4:GR WW5:GR WW6:GR WW7:GR WW1:GW WW2:GW  
## WeekfWeek2:GroupCONT   
## WeekfWeek3:GroupCONT   
## WeekfWeek4:GroupCONT   
## WeekfWeek5:GroupCONT   
## WeekfWeek6:GroupCONT   
## WeekfWeek7:GroupCONT   
## WeekfWeek1:GroupRI   
## WeekfWeek2:GroupRI   
## WeekfWeek3:GroupRI 0.952   
## WeekfWeek4:GroupRI 0.906 0.952   
## WeekfWeek5:GroupRI 0.862 0.906 0.952   
## WeekfWeek6:GroupRI 0.821 0.862 0.906 0.952   
## WeekfWeek7:GroupRI 0.781 0.821 0.862 0.906 0.952   
## WeekfWeek1:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000   
## WeekfWeek2:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.952   
## WeekfWeek3:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.906 0.952   
## WeekfWeek4:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.862 0.906   
## WeekfWeek5:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.821 0.862   
## WeekfWeek6:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.781 0.821   
## WeekfWeek7:GroupWI 0.000 0.000 0.000 0.000 0.000 0.000 0.743 0.781   
## WW3:GW WW4:GW WW5:GW WW6:GW  
## WeekfWeek2:GroupCONT   
## WeekfWeek3:GroupCONT   
## WeekfWeek4:GroupCONT   
## WeekfWeek5:GroupCONT   
## WeekfWeek6:GroupCONT   
## WeekfWeek7:GroupCONT   
## WeekfWeek1:GroupRI   
## WeekfWeek2:GroupRI   
## WeekfWeek3:GroupRI   
## WeekfWeek4:GroupRI   
## WeekfWeek5:GroupRI   
## WeekfWeek6:GroupRI   
## WeekfWeek7:GroupRI   
## WeekfWeek1:GroupWI   
## WeekfWeek2:GroupWI   
## WeekfWeek3:GroupWI   
## WeekfWeek4:GroupWI 0.952   
## WeekfWeek5:GroupWI 0.906 0.952   
## WeekfWeek6:GroupWI 0.862 0.906 0.952   
## WeekfWeek7:GroupWI 0.821 0.862 0.906 0.952   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.32270892 -0.62422802 -0.02903386 0.63874495 2.83080150   
##   
## Residual standard error: 3.280242   
## Degrees of freedom: 399 total; 378 residual

1. Toeplitz Structure (Model 4)

fit.toep\_1 <- glmmTMB(Strength ~ Group + Weekf + Weekf\*Group + toep(Weekf + 0 | IDf),   
 data = strength\_long1,   
 dispformula = ~ 0, # Assume homogeneity of covariance matrix across treatment groups  
 REML = TRUE)

## Warning in (function (start, objective, gradient = NULL, hessian = NULL, :  
## NA/NaN function evaluation  
## Warning in (function (start, objective, gradient = NULL, hessian = NULL, :  
## NA/NaN function evaluation

summary(fit.toep\_1)

## Family: gaussian ( identity )  
## Formula:   
## Strength ~ Group + Weekf + Weekf \* Group + toep(Weekf + 0 | IDf)  
## Dispersion: ~0  
## Data: strength\_long1  
##   
## AIC BIC logLik deviance df.resid   
## 1316.7 1452.3 -624.3 1248.7 365   
##   
## Random effects:  
##   
## Conditional model:  
## Groups Name Variance Std.Dev. Corr   
## IDf WeekfWeek1 8.750 2.958   
## WeekfWeek2 9.582 3.095 0.95   
## WeekfWeek3 10.852 3.294 0.91 0.95   
## WeekfWeek4 10.001 3.162 0.88 0.91 0.95   
## WeekfWeek5 12.054 3.472 0.85 0.88 0.91 0.95   
## WeekfWeek6 11.792 3.434 0.81 0.85 0.88 0.91 0.95   
## WeekfWeek7 12.561 3.544 0.80 0.81 0.85 0.88 0.91 0.95   
## Number of obs: 399, groups: IDf, 57  
##   
## Conditional model:  
## Estimate Std. Error z value Pr(>|z|)   
## (Intercept) 79.7500 0.6614 120.57 < 2e-16 \*\*\*  
## GroupRI -0.0625 0.9922 -0.06 0.949772   
## GroupWI 1.2976 0.9242 1.40 0.160320   
## WeekfWeek2 0.2000 0.2089 0.96 0.338468   
## WeekfWeek3 0.2500 0.2996 0.83 0.404095   
## WeekfWeek4 0.3000 0.3353 0.89 0.370953   
## WeekfWeek5 0.0500 0.4093 0.12 0.902777   
## WeekfWeek6 -0.1500 0.4490 -0.33 0.738302   
## WeekfWeek7 -0.1500 0.4744 -0.32 0.751887   
## GroupRI:WeekfWeek2 0.6750 0.3134 2.15 0.031264 \*   
## GroupWI:WeekfWeek2 0.4190 0.2919 1.44 0.151193   
## GroupRI:WeekfWeek3 0.8750 0.4495 1.95 0.051562 .   
## GroupWI:WeekfWeek3 0.6071 0.4187 1.45 0.147022   
## GroupRI:WeekfWeek4 1.0125 0.5030 2.01 0.044109 \*   
## GroupWI:WeekfWeek4 1.1762 0.4685 2.51 0.012059 \*   
## GroupRI:WeekfWeek5 1.5125 0.6140 2.46 0.013761 \*   
## GroupWI:WeekfWeek5 1.5214 0.5719 2.66 0.007810 \*\*   
## GroupRI:WeekfWeek6 1.5875 0.6734 2.36 0.018409 \*   
## GroupWI:WeekfWeek6 1.8167 0.6273 2.90 0.003781 \*\*   
## GroupRI:WeekfWeek7 1.5875 0.7117 2.23 0.025706 \*   
## GroupWI:WeekfWeek7 2.1976 0.6629 3.31 0.000917 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

fit.toep\_2 <- glmmTMB(Strength ~ -1 + Weekf:Group + toep(Weekf + 0 | IDf),   
 data = strength\_long1,   
 dispformula = ~0,   
 REML = TRUE)

## Warning in (function (start, objective, gradient = NULL, hessian = NULL, :  
## NA/NaN function evaluation

## Warning in (function (start, objective, gradient = NULL, hessian = NULL, :  
## NA/NaN function evaluation

## Warning in finalizeTMB(TMBStruc, obj, fit, h, data.tmb.old): Model convergence  
## problem; false convergence (8). See vignette('troubleshooting'),  
## help('diagnose')

summary(fit.toep\_2)

## Family: gaussian ( identity )  
## Formula: Strength ~ -1 + Weekf:Group + toep(Weekf + 0 | IDf)  
## Dispersion: ~0  
## Data: strength\_long1  
##   
## AIC BIC logLik deviance df.resid   
## 1316.7 1452.3 -624.3 1248.7 365   
##   
## Random effects:  
##   
## Conditional model:  
## Groups Name Variance Std.Dev. Corr   
## IDf WeekfWeek1 8.751 2.958   
## WeekfWeek2 9.582 3.095 0.95   
## WeekfWeek3 10.853 3.294 0.91 0.95   
## WeekfWeek4 10.002 3.163 0.88 0.91 0.95   
## WeekfWeek5 12.054 3.472 0.85 0.88 0.91 0.95   
## WeekfWeek6 11.792 3.434 0.81 0.85 0.88 0.91 0.95   
## WeekfWeek7 12.561 3.544 0.80 0.81 0.85 0.88 0.91 0.95   
## Number of obs: 399, groups: IDf, 57  
##   
## Conditional model:  
## Estimate Std. Error z value Pr(>|z|)   
## WeekfWeek1:GroupCONT 79.7500 0.6615 120.57 <2e-16 \*\*\*  
## WeekfWeek2:GroupCONT 79.9500 0.6922 115.51 <2e-16 \*\*\*  
## WeekfWeek3:GroupCONT 80.0000 0.7366 108.60 <2e-16 \*\*\*  
## WeekfWeek4:GroupCONT 80.0500 0.7072 113.20 <2e-16 \*\*\*  
## WeekfWeek5:GroupCONT 79.8000 0.7764 102.79 <2e-16 \*\*\*  
## WeekfWeek6:GroupCONT 79.6000 0.7679 103.67 <2e-16 \*\*\*  
## WeekfWeek7:GroupCONT 79.6000 0.7925 100.44 <2e-16 \*\*\*  
## WeekfWeek1:GroupRI 79.6875 0.7395 107.75 <2e-16 \*\*\*  
## WeekfWeek2:GroupRI 80.5625 0.7739 104.10 <2e-16 \*\*\*  
## WeekfWeek3:GroupRI 80.8125 0.8236 98.12 <2e-16 \*\*\*  
## WeekfWeek4:GroupRI 81.0000 0.7906 102.45 <2e-16 \*\*\*  
## WeekfWeek5:GroupRI 81.2500 0.8680 93.61 <2e-16 \*\*\*  
## WeekfWeek6:GroupRI 81.1250 0.8585 94.50 <2e-16 \*\*\*  
## WeekfWeek7:GroupRI 81.1250 0.8861 91.56 <2e-16 \*\*\*  
## WeekfWeek1:GroupWI 81.0476 0.6455 125.55 <2e-16 \*\*\*  
## WeekfWeek2:GroupWI 81.6667 0.6755 120.90 <2e-16 \*\*\*  
## WeekfWeek3:GroupWI 81.9048 0.7189 113.93 <2e-16 \*\*\*  
## WeekfWeek4:GroupWI 82.5238 0.6901 119.58 <2e-16 \*\*\*  
## WeekfWeek5:GroupWI 82.6190 0.7576 109.05 <2e-16 \*\*\*  
## WeekfWeek6:GroupWI 82.7143 0.7494 110.38 <2e-16 \*\*\*  
## WeekfWeek7:GroupWI 83.0952 0.7734 107.44 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

# Model Comparison: Extract AIC and BIC for Model Comparison

# Extract AIC, BIC, and log-likelihood values for each model  
aic\_values <- c(AIC(fit.unstructured\_1), AIC(fit.cs\_1), AIC(fit.ar1\_1), AIC(fit.toep\_1))  
bic\_values <- c(BIC(fit.unstructured\_1), BIC(fit.cs\_1), BIC(fit.ar1\_1), BIC(fit.toep\_1))  
logLik\_values <- c(logLik(fit.unstructured\_1), logLik(fit.cs\_1), logLik(fit.ar1\_1), logLik(fit.toep\_1))  
  
# Create a comparison data frame  
model\_names <- c("Unstructured Model 1", "Compound Symmetry Model 2", "AR(1) Model 3", "Toeplitz Model 4")  
Comparison\_Table <- data.frame(Model = model\_names,   
 AIC = aic\_values,   
 BIC = bic\_values,  
 LogLikelihood = logLik\_values)  
print(Comparison\_Table)

## Model AIC BIC LogLikelihood  
## 1 Unstructured Model 1 1332.896 1525.706 -617.4479  
## 2 Compound Symmetry Model 2 1466.820 1557.323 -710.4101  
## 3 AR(1) Model 3 1312.804 1403.306 -633.4018  
## 4 Toeplitz Model 4 1316.690 1452.315 -624.3451

* AR(1) Model 3 has the lowest AIC (1312.804) and BIC (1403.306), indicating it provides the best fit among the tested models when considering both criteria. Yet, Toeplitz Model 4 has the highest log-likelihood (-624.3451), meaning it fits the data better than the other models, even though it is penalized more by AIC and BIC for being more complex.

# To Perform Likelihood Ratio Tests:

# Likelihood ratio tests between Ar1 vs TOEPH  
# H0: AR(1), H1: TOEP   
df\_ar1 <- attr(logLik(fit.ar1\_1), "df")  
df\_toep <- attr(logLik(fit.toep\_1), "df")  
df\_difference <- df\_ar1 - df\_toep  
  
LRT\_statistic = -2 \* (logLik(fit.ar1\_1)-logLik(fit.toep\_1))  
p\_value <- pchisq(LRT\_statistic, df = abs(df\_difference), lower.tail = FALSE)  
# p\_value <- pchisq(18.1134, df = 11, lower.tail = FALSE)  
# p\_value  
  
# Print results  
cat("LRT\_statistic:", LRT\_statistic, "\n")

## LRT\_statistic: 18.11329

cat("Degrees of Freedom Difference:", abs(df\_difference), "\n")

## Degrees of Freedom Difference: 11

cat("P-value:", p\_value, "\n")

## P-value: 0.07898891

# P>0.05, thus H0 is not rejected and AR(1) is better model.

* From the log-likelihood ratio test, AR(1) model is sufficient, and there is no significant improvement in fit with the more complex Toeplitz model.

Conclusion: AR(1) Model is the most appropriate model for the data based on: 1) It has the lowest AIC and BIC, meaning it provides the best fit while balancing model complexity. 2) The LRT shows no significant improvement when using the more complex Toeplitz model, meaning AR(1) is sufficient for explaining the covariance structure.

Thus, all three —AIC, BIC, and the likelihood ratio test—are in agreement: AR(1) Model 3 is the most appropriate model for the data. The additional complexity of the Toeplitz model is not justified by a significant improvement in fit.

## Question3. Selected covariance model: AR(1) model.

1. Fit a model which includes effects of week, treatment, and their interaction (weekfxtreatment). Use GLS or glmmTMB, depending on the covariance structure. weekf is week as a factor.

fit.ar1\_1 <- gls(Strength ~ Group + Weekf + Weekf\*Group,   
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.ar1\_1)

## Generalized least squares fit by REML  
## Model: Strength ~ Group + Weekf + Weekf \* Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1312.804 1403.306 -633.4018  
##   
## Correlation Structure: AR(1)  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Phi   
## 0.9517769   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## (Intercept) 79.75000 0.7334844 108.72760 0.0000  
## GroupRI -0.06250 1.1002267 -0.05681 0.9547  
## GroupWI 1.29762 1.0248804 1.26612 0.2063  
## WeekfWeek2 0.20000 0.2277894 0.87800 0.3805  
## WeekfWeek3 0.25000 0.3182354 0.78558 0.4326  
## WeekfWeek4 0.30000 0.3850689 0.77908 0.4364  
## WeekfWeek5 0.05000 0.4393354 0.11381 0.9095  
## WeekfWeek6 -0.15000 0.4853821 -0.30903 0.7575  
## WeekfWeek7 -0.15000 0.5254737 -0.28546 0.7755  
## GroupRI:WeekfWeek2 0.67500 0.3416840 1.97551 0.0489  
## GroupWI:WeekfWeek2 0.41905 0.3182847 1.31658 0.1888  
## GroupRI:WeekfWeek3 0.87500 0.4773531 1.83302 0.0676  
## GroupWI:WeekfWeek3 0.60714 0.4446628 1.36540 0.1729  
## GroupRI:WeekfWeek4 1.01250 0.5776033 1.75293 0.0804  
## GroupWI:WeekfWeek4 1.17619 0.5380476 2.18603 0.0294  
## GroupRI:WeekfWeek5 1.51250 0.6590031 2.29513 0.0223  
## GroupWI:WeekfWeek5 1.52143 0.6138730 2.47841 0.0136  
## GroupRI:WeekfWeek6 1.58750 0.7280732 2.18041 0.0298  
## GroupWI:WeekfWeek6 1.81667 0.6782129 2.67861 0.0077  
## GroupRI:WeekfWeek7 1.58750 0.7882106 2.01406 0.0447  
## GroupWI:WeekfWeek7 2.19762 0.7342320 2.99309 0.0029  
##   
## Correlation:   
## (Intr) GropRI GropWI WkfWk2 WkfWk3 WkfWk4 WkfWk5 WkfWk6  
## GroupRI -0.667   
## GroupWI -0.716 0.477   
## WeekfWeek2 -0.155 0.104 0.111   
## WeekfWeek3 -0.217 0.145 0.155 0.699   
## WeekfWeek4 -0.262 0.175 0.188 0.564 0.807   
## WeekfWeek5 -0.299 0.200 0.214 0.483 0.690 0.855   
## WeekfWeek6 -0.331 0.221 0.237 0.427 0.610 0.756 0.883   
## WeekfWeek7 -0.358 0.239 0.256 0.386 0.551 0.682 0.797 0.901  
## GroupRI:WeekfWeek2 0.104 -0.155 -0.074 -0.667 -0.466 -0.376 -0.322 -0.285  
## GroupWI:WeekfWeek2 0.111 -0.074 -0.155 -0.716 -0.500 -0.403 -0.346 -0.306  
## GroupRI:WeekfWeek3 0.145 -0.217 -0.104 -0.466 -0.667 -0.538 -0.460 -0.407  
## GroupWI:WeekfWeek3 0.155 -0.104 -0.217 -0.500 -0.716 -0.577 -0.494 -0.437  
## GroupRI:WeekfWeek4 0.175 -0.262 -0.125 -0.376 -0.538 -0.667 -0.570 -0.504  
## GroupWI:WeekfWeek4 0.188 -0.125 -0.262 -0.403 -0.577 -0.716 -0.612 -0.541  
## GroupRI:WeekfWeek5 0.200 -0.299 -0.143 -0.322 -0.460 -0.570 -0.667 -0.589  
## GroupWI:WeekfWeek5 0.214 -0.143 -0.299 -0.346 -0.494 -0.612 -0.716 -0.632  
## GroupRI:WeekfWeek6 0.221 -0.331 -0.158 -0.285 -0.407 -0.504 -0.589 -0.667  
## GroupWI:WeekfWeek6 0.237 -0.158 -0.331 -0.306 -0.437 -0.541 -0.632 -0.716  
## GroupRI:WeekfWeek7 0.239 -0.358 -0.171 -0.257 -0.368 -0.455 -0.531 -0.601  
## GroupWI:WeekfWeek7 0.256 -0.171 -0.358 -0.276 -0.395 -0.488 -0.570 -0.645  
## WkfWk7 GRI:WW2 GWI:WW2 GRI:WW3 GWI:WW3 GRI:WW4 GWI:WW4  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2 -0.257   
## GroupWI:WeekfWeek2 -0.276 0.477   
## GroupRI:WeekfWeek3 -0.368 0.699 0.333   
## GroupWI:WeekfWeek3 -0.395 0.333 0.699 0.477   
## GroupRI:WeekfWeek4 -0.455 0.564 0.269 0.807 0.385   
## GroupWI:WeekfWeek4 -0.488 0.269 0.564 0.385 0.807 0.477   
## GroupRI:WeekfWeek5 -0.531 0.483 0.230 0.690 0.329 0.855 0.408   
## GroupWI:WeekfWeek5 -0.570 0.230 0.483 0.329 0.690 0.408 0.855   
## GroupRI:WeekfWeek6 -0.601 0.427 0.204 0.610 0.291 0.756 0.361   
## GroupWI:WeekfWeek6 -0.645 0.204 0.427 0.291 0.610 0.361 0.756   
## GroupRI:WeekfWeek7 -0.667 0.386 0.184 0.551 0.263 0.682 0.326   
## GroupWI:WeekfWeek7 -0.716 0.184 0.386 0.263 0.551 0.326 0.682   
## GRI:WW5 GWI:WW5 GRI:WW6 GWI:WW6 GRI:WW7  
## GroupRI   
## GroupWI   
## WeekfWeek2   
## WeekfWeek3   
## WeekfWeek4   
## WeekfWeek5   
## WeekfWeek6   
## WeekfWeek7   
## GroupRI:WeekfWeek2   
## GroupWI:WeekfWeek2   
## GroupRI:WeekfWeek3   
## GroupWI:WeekfWeek3   
## GroupRI:WeekfWeek4   
## GroupWI:WeekfWeek4   
## GroupRI:WeekfWeek5   
## GroupWI:WeekfWeek5 0.477   
## GroupRI:WeekfWeek6 0.883 0.421   
## GroupWI:WeekfWeek6 0.421 0.883 0.477   
## GroupRI:WeekfWeek7 0.797 0.380 0.901 0.430   
## GroupWI:WeekfWeek7 0.380 0.797 0.430 0.901 0.477   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.32270892 -0.62422802 -0.02903386 0.63874495 2.83080150   
##   
## Residual standard error: 3.280242   
## Degrees of freedom: 399 total; 378 residual

anova(fit.ar1\_1)

## Denom. DF: 378   
## numDF F-value p-value  
## (Intercept) 1 39707.71 <.0001  
## Group 2 3.27 0.0390  
## Weekf 6 4.22 0.0004  
## Group:Weekf 12 1.17 0.3000

Is treatment\*weekf significant?

No. P-value for the interaction is 0.03 > 0.05. Thus, the interaction between group and weekf is not statistically significant, implying that the group (treatment) effect on strength does not significantly differ across the weeks.

At which weeks is the difference among the treatment groups significant?

The sitfinificant differences occured mostly in the later weeks. To be specific, at week 5,6, and 7, both the RI and WI group show significant differences from the control group with p<0.05. Significant differences between RI and the control group occur at Weeks 2, 5, 6, and 7. Significant differences between WI and the control group occur at Weeks 4, 5, 6, and 7.

1. Use the nested model without grand mean (-1 + weekf:treatment). Use orthogonal polynomials contrasts to test linear and quadratic trends in the WI treatment group and the RI group. Write down estimates of the trends and test statistics. Which trends are significant?

fit.ar1\_2 <- gls(Strength ~ -1 + Weekf:Group,   
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "REML")  
#summary(fit.ar1\_2)  
  
  
# Test polynormial trends using orthogonal polynomial contrast coefficients:  
fit.ar1\_2$coefficients

## WeekfWeek1:GroupCONT WeekfWeek2:GroupCONT WeekfWeek3:GroupCONT   
## 79.75000 79.95000 80.00000   
## WeekfWeek4:GroupCONT WeekfWeek5:GroupCONT WeekfWeek6:GroupCONT   
## 80.05000 79.80000 79.60000   
## WeekfWeek7:GroupCONT WeekfWeek1:GroupRI WeekfWeek2:GroupRI   
## 79.60000 79.68750 80.56250   
## WeekfWeek3:GroupRI WeekfWeek4:GroupRI WeekfWeek5:GroupRI   
## 80.81250 81.00000 81.25000   
## WeekfWeek6:GroupRI WeekfWeek7:GroupRI WeekfWeek1:GroupWI   
## 81.12500 81.12500 81.04762   
## WeekfWeek2:GroupWI WeekfWeek3:GroupWI WeekfWeek4:GroupWI   
## 81.66667 81.90476 82.52381   
## WeekfWeek5:GroupWI WeekfWeek6:GroupWI WeekfWeek7:GroupWI   
## 82.61905 82.71429 83.09524

# contrast  
contr.poly(7)

## .L .Q .C ^4 ^5  
## [1,] -5.669467e-01 5.455447e-01 -4.082483e-01 0.2417469 -1.091089e-01  
## [2,] -3.779645e-01 9.690821e-17 4.082483e-01 -0.5640761 4.364358e-01  
## [3,] -1.889822e-01 -3.273268e-01 4.082483e-01 0.0805823 -5.455447e-01  
## [4,] 2.098124e-17 -4.364358e-01 4.532467e-17 0.4834938 5.342065e-16  
## [5,] 1.889822e-01 -3.273268e-01 -4.082483e-01 0.0805823 5.455447e-01  
## [6,] 3.779645e-01 0.000000e+00 -4.082483e-01 -0.5640761 -4.364358e-01  
## [7,] 5.669467e-01 5.455447e-01 4.082483e-01 0.2417469 1.091089e-01  
## ^6  
## [1,] 0.03289758  
## [2,] -0.19738551  
## [3,] 0.49346377  
## [4,] -0.65795169  
## [5,] 0.49346377  
## [6,] -0.19738551  
## [7,] 0.03289758

orthpoly <- contr.poly(7)  
zerovec <- c (0,0,0,0,0,0,0)

**To test linear and quadratic trends in the RI group:**

# Contrast coeff for linear trend in RI group  
c1\_RI <- c( zerovec, orthpoly[,1], zerovec)  
c1\_RI

## [1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00  
## [6] 0.000000e+00 0.000000e+00 -5.669467e-01 -3.779645e-01 -1.889822e-01  
## [11] 2.098124e-17 1.889822e-01 3.779645e-01 5.669467e-01 0.000000e+00  
## [16] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00  
## [21] 0.000000e+00

betahat <- fit.ar1\_2$coefficients # extract coefficiennts  
c1\_RI%%betahat

## WeekfWeek1:GroupCONT WeekfWeek2:GroupCONT WeekfWeek3:GroupCONT   
## 0.000000e+00 0.000000e+00 0.000000e+00   
## WeekfWeek4:GroupCONT WeekfWeek5:GroupCONT WeekfWeek6:GroupCONT   
## 0.000000e+00 0.000000e+00 0.000000e+00   
## WeekfWeek7:GroupCONT WeekfWeek1:GroupRI WeekfWeek2:GroupRI   
## 0.000000e+00 7.912055e+01 8.018454e+01   
## WeekfWeek3:GroupRI WeekfWeek4:GroupRI WeekfWeek5:GroupRI   
## 8.062352e+01 2.098124e-17 1.889822e-01   
## WeekfWeek6:GroupRI WeekfWeek7:GroupRI WeekfWeek1:GroupWI   
## 3.779645e-01 5.669467e-01 0.000000e+00   
## WeekfWeek2:GroupWI WeekfWeek3:GroupWI WeekfWeek4:GroupWI   
## 0.000000e+00 0.000000e+00 0.000000e+00   
## WeekfWeek5:GroupWI WeekfWeek6:GroupWI WeekfWeek7:GroupWI   
## 0.000000e+00 0.000000e+00 0.000000e+00

linear\_trend\_RI <- sum(c1\_RI \* betahat) # Estimate the linear trend for RI  
cat("Est.linear\_trend\_RI:", linear\_trend\_RI, "\n")

## Est.linear\_trend\_RI: 1.110271

# Quadratic contrast vector for RI group  
c2\_RI <- c(zerovec, orthpoly[,2], zerovec) # Contrast for RI quadratic trend  
quadratic\_trend\_RI <- sum(c2\_RI \* betahat) # Estimate the quadratic trend for RI  
cat("Est.quadratic\_trend\_RI:", quadratic\_trend\_RI, "\n")

## Est.quadratic\_trend\_RI: -0.6682923

**To test linear and quadratic trends in the WI group:**

# Contrast coeff for linear trend in WI group  
c1\_WI <- c( zerovec, zerovec, orthpoly[,1])  
c1\_WI

## [1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00  
## [6] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00  
## [11] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 -5.669467e-01  
## [16] -3.779645e-01 -1.889822e-01 2.098124e-17 1.889822e-01 3.779645e-01  
## [21] 5.669467e-01

c1\_WI%%betahat

## WeekfWeek1:GroupCONT WeekfWeek2:GroupCONT WeekfWeek3:GroupCONT   
## 0.000000e+00 0.000000e+00 0.000000e+00   
## WeekfWeek4:GroupCONT WeekfWeek5:GroupCONT WeekfWeek6:GroupCONT   
## 0.000000e+00 0.000000e+00 0.000000e+00   
## WeekfWeek7:GroupCONT WeekfWeek1:GroupRI WeekfWeek2:GroupRI   
## 0.000000e+00 0.000000e+00 0.000000e+00   
## WeekfWeek3:GroupRI WeekfWeek4:GroupRI WeekfWeek5:GroupRI   
## 0.000000e+00 0.000000e+00 0.000000e+00   
## WeekfWeek6:GroupRI WeekfWeek7:GroupRI WeekfWeek1:GroupWI   
## 0.000000e+00 0.000000e+00 8.048067e+01   
## WeekfWeek2:GroupWI WeekfWeek3:GroupWI WeekfWeek4:GroupWI   
## 8.128870e+01 8.171578e+01 2.098124e-17   
## WeekfWeek5:GroupWI WeekfWeek6:GroupWI WeekfWeek7:GroupWI   
## 1.889822e-01 3.779645e-01 5.669467e-01

linear\_trend\_WI <- sum(c1\_WI \* betahat) # Estimate the linear trend for WI  
cat("Est.linear\_trend\_WI:", linear\_trend\_WI, "\n")

## Est.linear\_trend\_WI: 1.691841

# Quadratic contrast vector for RI group  
c2\_WI <- c(zerovec, zerovec, orthpoly[,2]) # Contrast for WI quadratic trend  
quadratic\_trend\_WI <- sum(c2\_WI \* betahat) # Estimate the quadratic trend for WI  
cat("Est.quadratic\_trend\_WI:", quadratic\_trend\_WI, "\n")

## Est.quadratic\_trend\_WI: -0.3221312

**T statistics and p-value for linear trend for RI:**

# Get covariance matrix of the model  
covmat <- vcov(fit.ar1\_2)  
  
# Calculate the standard error of the linear trend for RI  
linear\_trend\_RI\_se <- sqrt(t(c1\_RI) %\*% covmat %\*% c1\_RI)  
  
cat("linear\_trend\_RI:",linear\_trend\_RI, "\n")

## linear\_trend\_RI: 1.110271

cat("linear\_trend\_RI\_se:",linear\_trend\_RI\_se, "\n")

## linear\_trend\_RI\_se: 0.5386385

# Calculate t-value and p-value for the linear trend in RI  
t\_value\_linear\_RI <- linear\_trend\_RI / linear\_trend\_RI\_se  
  
# df  
N <- nrow(strength\_long1)  
p <- length(fit.ar1\_2$coefficients)  
df\_residual <- N - p  
cat("df\_residual:", df\_residual,"\n") # 378

## df\_residual: 378

p\_value\_linear\_RI <- 2 \* pt(-abs(t\_value\_linear\_RI), df\_residual)  
  
  
cat("t statistic:",t\_value\_linear\_RI, "\n")

## t statistic: 2.061254

cat("p-value:",p\_value\_linear\_RI, "\n")

## p-value: 0.03996226

=> linear\_trend\_RI is significant (P<0.05).

# Calculate the standard error of the linear trend for WI  
linear\_trend\_WI\_se <- sqrt(t(c1\_WI) %\*% covmat %\*% c1\_WI)  
  
cat("linear\_trend\_WI:",linear\_trend\_WI, "\n")

## linear\_trend\_WI: 1.691841

cat("linear\_trend\_WI\_se:",linear\_trend\_WI\_se, "\n")

## linear\_trend\_WI\_se: 0.4701622

# Calculate t-value and p-value for the linear trend in RI  
t\_value\_linear\_WI <- linear\_trend\_WI / linear\_trend\_WI\_se  
  
p\_value\_linear\_WI <- 2 \* pt(-abs(t\_value\_linear\_WI), df\_residual)  
  
  
cat("t statistic:",t\_value\_linear\_WI, "\n")

## t statistic: 3.598419

cat("p-value:",p\_value\_linear\_WI, "\n")

## p-value: 0.0003627349

=> linear\_trend\_WI is significant (P>0.05).

# Calculate the standard error of the quadratic trend for RI  
quadratic\_trend\_RI\_se <- sqrt(t(c2\_RI) %\*% covmat %\*% c2\_RI)  
  
cat("quadratic\_trend\_RI:",quadratic\_trend\_RI, "\n")

## quadratic\_trend\_RI: -0.6682923

cat("quadratic\_trend\_RI\_se:",quadratic\_trend\_RI\_se, "\n")

## quadratic\_trend\_RI\_se: 0.2912402

# Calculate t-value and p-value for the quadratic trend in RI  
t\_value\_quadratic\_RI <- quadratic\_trend\_RI / quadratic\_trend\_RI\_se  
  
p\_value\_quadratic\_RI <- 2 \* pt(-abs(t\_value\_quadratic\_RI), df\_residual)  
  
  
cat("t statistic:",t\_value\_quadratic\_RI, "\n")

## t statistic: -2.294643

cat("p-value:",p\_value\_quadratic\_RI, "\n")

## p-value: 0.02230101

=> quadratic\_trend\_RI is significant (P<0.05)

# Calculate the standard error of the quadratic trend for WI  
quadratic\_trend\_WI\_se <- sqrt(t(c2\_WI) %\*% covmat %\*% c2\_WI)  
  
cat("quadratic\_trend\_WI:",quadratic\_trend\_WI, "\n")

## quadratic\_trend\_WI: -0.3221312

cat("quadratic\_trend\_WI\_se:",quadratic\_trend\_WI\_se, "\n")

## quadratic\_trend\_WI\_se: 0.2542153

# Calculate t-value and p-value for the quadratic trend in RI  
t\_value\_quadratic\_WI <- quadratic\_trend\_WI / quadratic\_trend\_WI\_se  
  
p\_value\_quadratic\_WI <- 2 \* pt(-abs(t\_value\_quadratic\_WI), df\_residual)  
  
  
cat("t statistic:",t\_value\_quadratic\_WI, "\n")

## t statistic: -1.267159

cat("p-value:",p\_value\_quadratic\_WI, "\n")

## p-value: 0.2058786

=> quadratic\_trend\_WI is not significant (P>0.05).

# Create a dataframe to display the results using the notation from your code  
trend\_results <- data.frame(  
 Group = c("RI", "RI", "WI", "WI"),  
 Trend = c("Linear", "Quadratic", "Linear", "Quadratic"),  
 Estimate = c(linear\_trend\_RI, quadratic\_trend\_RI, linear\_trend\_WI, quadratic\_trend\_WI),  
 `Standard Error` = c(linear\_trend\_RI\_se, quadratic\_trend\_RI\_se, linear\_trend\_WI\_se, quadratic\_trend\_WI\_se),  
 `t-value` = c(t\_value\_linear\_RI, t\_value\_quadratic\_RI, t\_value\_linear\_WI, t\_value\_quadratic\_WI),  
 `p-value` = c(p\_value\_linear\_RI, p\_value\_quadratic\_RI, p\_value\_linear\_WI, p\_value\_quadratic\_WI)  
)  
  
# Display the dataframe as a table  
print(trend\_results)

## Group Trend Estimate Standard.Error t.value p.value  
## 1 RI Linear 1.1102706 0.5386385 2.061254 0.0399622634  
## 2 RI Quadratic -0.6682923 0.2912402 -2.294643 0.0223010083  
## 3 WI Linear 1.6918410 0.4701622 3.598419 0.0003627349  
## 4 WI Quadratic -0.3221312 0.2542153 -1.267159 0.2058786212

The linear trend is significant for both the RI and WI groups, indicating that strength increases over time for both treatment groups. The quadratic trend is significant only for the RI group, suggesting that strength improvement in the RI group is non-linear, while the WI group shows no significant quadratic trend.

1. Use a contrast with orthogonal polynomial coefficients to test the null hypothesis that the quadratic trend for WI is equal to the quadratic trend for RI. This is a contrast of contrasts. Write down estimate of contrast and test statistic. Is there a significant difference? Why or why not?

H0: beta\_quadratic\_WI = beta\_quadatic RI

H1: beta\_quadratic\_WI ≠ beta\_quadatic RI

c2\_contrast <- c2\_WI - c2\_RI  
# Estimate the contrast (quadratic trend WI - quadratic trend RI)  
contrast\_estimate <- sum(c2\_contrast \* betahat)  
cat("Estimate of contrast:", contrast\_estimate, "\n")

## Estimate of contrast: 0.3461611

# Standard error of the contrast  
contrast\_se <- sqrt(t(c2\_contrast) %\*% covmat %\*% c2\_contrast)  
cat("Standard error of contrast:", contrast\_se, "\n")

## Standard error of contrast: 0.3865828

# Calculate the t-value for the contrast  
t\_value\_contrast <- contrast\_estimate / contrast\_se  
cat("t-value for contrast:", t\_value\_contrast, "\n")

## t-value for contrast: 0.8954384

# Calculate the p-value for the contrast  
p\_value\_contrast <- 2 \* pt(-abs(t\_value\_contrast), df\_residual)  
cat("p-value for contrast:", p\_value\_contrast, "\n")

## p-value for contrast: 0.3711226

With p=0.3711226 > 0.05, there is no significant difference between the quadratic trends for the WI and RI groups. Thus, we fail to reject the null hypothesis. The data does not provide enough evidence to conclude that the rate of non-linear (quadratic) change in strength over time differs between the two groups.

## **Question 4. Selected model (AR(1) with continuous week.**

1. Estimate a model with linear, quadratic and cubic trends over time and interaction of polynomial trends by treatment group. Test weekxtreatment, weeksqxtreatment and weekcubedxtreatment.

strength\_long1$Week\_sq <- strength\_long1$Week\_num^2  
strength\_long1$Week\_cub <- strength\_long1$Week\_num^3  
  
fit.ar1\_poly <- gls(Strength ~ Group + Week\_num + Week\_sq + Week\_cub +  
 Week\_num\*Group + Week\_sq\*Group + Week\_cub\*Group,   
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.ar1\_poly)

## Generalized least squares fit by REML  
## Model: Strength ~ Group + Week\_num + Week\_sq + Week\_cub + Week\_num \* Group + Week\_sq \* Group + Week\_cub \* Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1326.86 1382.278 -649.4299  
##   
## Correlation Structure: AR(1)  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Phi   
## 0.9522391   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## (Intercept) 79.24492 0.8711288 90.96808 0.0000  
## GroupRI -0.75720 1.3066932 -0.57947 0.5626  
## GroupWI 0.94558 1.2172076 0.77684 0.4377  
## Week\_num 0.66204 0.5417672 1.22200 0.2225  
## Week\_sq -0.16858 0.1525413 -1.10511 0.2698  
## Week\_cub 0.01161 0.0125062 0.92805 0.3540  
## GroupRI:Week\_num 0.77906 0.8126507 0.95866 0.3383  
## GroupWI:Week\_num 0.34413 0.7569984 0.45460 0.6497  
## GroupRI:Week\_sq -0.08761 0.2288120 -0.38289 0.7020  
## GroupWI:Week\_sq 0.00875 0.2131424 0.04106 0.9673  
## GroupRI:Week\_cub 0.00327 0.0187593 0.17433 0.8617  
## GroupWI:Week\_cub -0.00084 0.0174746 -0.04806 0.9617  
##   
## Correlation:   
## (Intr) GropRI GropWI Wek\_nm Wek\_sq Wek\_cb GrpRI:Wk\_n  
## GroupRI -0.667   
## GroupWI -0.716 0.477   
## Week\_num -0.526 0.350 0.376   
## Week\_sq 0.440 -0.293 -0.315 -0.958   
## Week\_cub -0.403 0.269 0.288 0.901 -0.984   
## GroupRI:Week\_num 0.350 -0.526 -0.251 -0.667 0.639 -0.600   
## GroupWI:Week\_num 0.376 -0.251 -0.526 -0.716 0.686 -0.645 0.477   
## GroupRI:Week\_sq -0.293 0.440 0.210 0.639 -0.667 0.656 -0.958   
## GroupWI:Week\_sq -0.315 0.210 0.440 0.686 -0.716 0.704 -0.457   
## GroupRI:Week\_cub 0.269 -0.403 -0.192 -0.600 0.656 -0.667 0.901   
## GroupWI:Week\_cub 0.288 -0.192 -0.403 -0.645 0.704 -0.716 0.430   
## GrpWI:Wk\_n GrpRI:Wk\_s GrpWI:Wk\_s GrpRI:Wk\_c  
## GroupRI   
## GroupWI   
## Week\_num   
## Week\_sq   
## Week\_cub   
## GroupRI:Week\_num   
## GroupWI:Week\_num   
## GroupRI:Week\_sq -0.457   
## GroupWI:Week\_sq -0.958 0.477   
## GroupRI:Week\_cub 0.430 -0.984 -0.469   
## GroupWI:Week\_cub 0.901 -0.469 -0.984 0.477   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.3083393 -0.6242544 -0.0290322 0.6209849 2.8049798   
##   
## Residual standard error: 3.280089   
## Degrees of freedom: 399 total; 387 residual

anova(fit.ar1\_poly)

## Denom. DF: 387   
## numDF F-value p-value  
## (Intercept) 1 39660.81 <.0001  
## Group 2 3.27 0.0391  
## Week\_num 1 12.68 0.0004  
## Week\_sq 1 7.18 0.0077  
## Week\_cub 1 2.72 0.1000  
## Group:Week\_num 2 4.74 0.0092  
## Group:Week\_sq 2 0.88 0.4168  
## Group:Week\_cub 2 0.03 0.9740

Which polynomial trends differ by treatment group? Use the model for ΣY selected above.

=> Week\_num and Week\_sq: There is significant linear and quadratic trend over time across all groups. Cubic trend is NOT significant.

Group\*Week\_num : Linear trend by group is significantly different, meaning the rate of linear change in strength over time is significantly different among WI, RI, and CONT groups. Quadratic or cubic trend by groups are not significantly different.

weeks <- 1:7  
  
# Define the treatment group (0 for CONT group)  
# Since this is for the control group, we don't include treatment terms  
X\_i\_control <- cbind(  
 Intct = rep(1, length(weeks)), # Intercept (β0)  
 Linear = weeks, # Linear time effect (β1 \* t)  
 Quad = weeks^2, # Quadratic time effect (β2 \* t^2)  
 Cubic = weeks^3, # Cubic time effect (β3 \* t^3)  
 RI = rep(0, length(weeks)), # No treatment 1, RI  
 WI = rep(0, length(weeks)), # No treatment 2, WI   
 L\_RI = rep(0, length(weeks)), # No linear interaction with trt 1  
 L\_WI = rep(0, length(weeks)), # No linear interaction with trt 2  
 Q\_RI = rep(0, length(weeks)), # No quadratic interaction with trt 1  
 Q\_WI = rep(0, length(weeks)), # No quadratic interaction with trt 2  
 C\_RI = rep(0, length(weeks)), # No cubic interaction with trt 1  
 C\_WI = rep(0, length(weeks)) # No cubic interaction with trt 2  
)  
  
# Print the Xi matrix for the control group  
print(X\_i\_control)

## Intct Linear Quad Cubic RI WI L\_RI L\_WI Q\_RI Q\_WI C\_RI C\_WI  
## [1,] 1 1 1 1 0 0 0 0 0 0 0 0  
## [2,] 1 2 4 8 0 0 0 0 0 0 0 0  
## [3,] 1 3 9 27 0 0 0 0 0 0 0 0  
## [4,] 1 4 16 64 0 0 0 0 0 0 0 0  
## [5,] 1 5 25 125 0 0 0 0 0 0 0 0  
## [6,] 1 6 36 216 0 0 0 0 0 0 0 0  
## [7,] 1 7 49 343 0 0 0 0 0 0 0 0

**(d)use the nested model (-1 + week:treatment). Include week, weeksq, and weekcubed. Which trends are significant for CONT? for RI? for WI?**

fit.ar1\_poly\_nested <-   
 gls(Strength ~ -1 + Group + Week\_num:Group + Week\_sq:Group + Week\_cub:Group,  
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.ar1\_poly\_nested)

## Generalized least squares fit by REML  
## Model: Strength ~ -1 + Group + Week\_num:Group + Week\_sq:Group + Week\_cub:Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1326.86 1382.278 -649.4299  
##   
## Correlation Structure: AR(1)  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Phi   
## 0.9522391   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## GroupCONT 79.24492 0.8711288 90.96808 0.0000  
## GroupRI 78.48772 0.9739516 80.58688 0.0000  
## GroupWI 80.19050 0.8501347 94.32682 0.0000  
## GroupCONT:Week\_num 0.66204 0.5417672 1.22200 0.2225  
## GroupRI:Week\_num 1.44110 0.6057141 2.37917 0.0178  
## GroupWI:Week\_num 1.00617 0.5287106 1.90306 0.0578  
## GroupCONT:Week\_sq -0.16858 0.1525413 -1.10511 0.2698  
## GroupRI:Week\_sq -0.25619 0.1705464 -1.50215 0.1339  
## GroupWI:Week\_sq -0.15982 0.1488651 -1.07362 0.2837  
## GroupCONT:Week\_cub 0.01161 0.0125062 0.92805 0.3540  
## GroupRI:Week\_cub 0.01488 0.0139824 1.06397 0.2880  
## GroupWI:Week\_cub 0.01077 0.0122048 0.88216 0.3782  
##   
## Correlation:   
## GrCONT GropRI GropWI GrpCONT:Wk\_n GrpRI:Wk\_n GrpWI:Wk\_n  
## GroupRI 0.000   
## GroupWI 0.000 0.000   
## GroupCONT:Week\_num -0.526 0.000 0.000   
## GroupRI:Week\_num 0.000 -0.526 0.000 0.000   
## GroupWI:Week\_num 0.000 0.000 -0.526 0.000 0.000   
## GroupCONT:Week\_sq 0.440 0.000 0.000 -0.958 0.000 0.000   
## GroupRI:Week\_sq 0.000 0.440 0.000 0.000 -0.958 0.000   
## GroupWI:Week\_sq 0.000 0.000 0.440 0.000 0.000 -0.958   
## GroupCONT:Week\_cub -0.403 0.000 0.000 0.901 0.000 0.000   
## GroupRI:Week\_cub 0.000 -0.403 0.000 0.000 0.901 0.000   
## GroupWI:Week\_cub 0.000 0.000 -0.403 0.000 0.000 0.901   
## GrpCONT:Wk\_s GrpRI:Wk\_s GrpWI:Wk\_s GrpCONT:Wk\_c GrpRI:Wk\_c  
## GroupRI   
## GroupWI   
## GroupCONT:Week\_num   
## GroupRI:Week\_num   
## GroupWI:Week\_num   
## GroupCONT:Week\_sq   
## GroupRI:Week\_sq 0.000   
## GroupWI:Week\_sq 0.000 0.000   
## GroupCONT:Week\_cub -0.984 0.000 0.000   
## GroupRI:Week\_cub 0.000 -0.984 0.000 0.000   
## GroupWI:Week\_cub 0.000 0.000 -0.984 0.000 0.000   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.3083393 -0.6242544 -0.0290322 0.6209849 2.8049798   
##   
## Residual standard error: 3.280089   
## Degrees of freedom: 399 total; 387 residual

anova(fit.ar1\_poly\_nested)

## Denom. DF: 387   
## numDF F-value p-value  
## Group 3 13222.450 <.0001  
## Group:Week\_num 3 7.390 0.0001  
## Group:Week\_sq 3 2.978 0.0315  
## Group:Week\_cub 3 0.924 0.4293

**Center week to reduce collinearity.**

average\_week <- mean(1:7)  
average\_week #4

## [1] 4

# Center the week (average\_week = 4)  
strength\_long1$Week\_c <- strength\_long1$Week\_num - 4  
strength\_long1$Week\_c\_sq <- strength\_long1$Week\_c\*strength\_long1$Week\_c  
strength\_long1$Week\_c\_cub <- strength\_long1$Week\_c\*strength\_long1$Week\_c\_sq  
  
fit.ar1\_poly\_nested\_centered <-gls(Strength ~   
 -1 + Group + Week\_c:Group + Week\_c\_sq:Group + Week\_c\_cub:Group,  
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "REML")  
summary(fit.ar1\_poly\_nested\_centered)

## Generalized least squares fit by REML  
## Model: Strength ~ -1 + Group + Week\_c:Group + Week\_c\_sq:Group + Week\_c\_cub:Group   
## Data: strength\_long1   
## AIC BIC logLik  
## 1326.86 1382.278 -649.4299  
##   
## Correlation Structure: AR(1)  
## Formula: ~1 | IDf   
## Parameter estimate(s):  
## Phi   
## 0.9522391   
##   
## Coefficients:  
## Value Std.Error t-value p-value  
## GroupCONT 79.93868 0.7210719 110.86090 0.0000  
## GroupRI 81.10525 0.8061829 100.60403 0.0000  
## GroupWI 82.34705 0.7036941 117.02108 0.0000  
## GroupCONT:Week\_c -0.12946 0.1421929 -0.91044 0.3632  
## GroupRI:Week\_c 0.10569 0.1589765 0.66483 0.5066  
## GroupWI:Week\_c 0.24437 0.1387661 1.76102 0.0790  
## GroupCONT:Week\_c\_sq -0.02930 0.0273230 -1.07232 0.2842  
## GroupRI:Week\_c\_sq -0.07766 0.0305480 -2.54239 0.0114  
## GroupWI:Week\_c\_sq -0.03063 0.0266645 -1.14854 0.2515  
## GroupCONT:Week\_c\_cub 0.01161 0.0125062 0.92805 0.3540  
## GroupRI:Week\_c\_cub 0.01488 0.0139824 1.06397 0.2880  
## GroupWI:Week\_c\_cub 0.01077 0.0122048 0.88216 0.3782  
##   
## Correlation:   
## GrCONT GropRI GropWI GrCONT:W\_ GrRI:W\_ GrWI:W\_  
## GroupRI 0.000   
## GroupWI 0.000 0.000   
## GroupCONT:Week\_c 0.000 0.000 0.000   
## GroupRI:Week\_c 0.000 0.000 0.000 0.000   
## GroupWI:Week\_c 0.000 0.000 0.000 0.000 0.000   
## GroupCONT:Week\_c\_sq -0.313 0.000 0.000 0.000 0.000 0.000   
## GroupRI:Week\_c\_sq 0.000 -0.313 0.000 0.000 0.000 0.000   
## GroupWI:Week\_c\_sq 0.000 0.000 -0.313 0.000 0.000 0.000   
## GroupCONT:Week\_c\_cub 0.000 0.000 0.000 -0.790 0.000 0.000   
## GroupRI:Week\_c\_cub 0.000 0.000 0.000 0.000 -0.790 0.000   
## GroupWI:Week\_c\_cub 0.000 0.000 0.000 0.000 0.000 -0.790   
## GrpCONT:Wk\_c\_s GrpRI:Wk\_c\_s GrpWI:Wk\_c\_s GrpCONT:Wk\_c\_c  
## GroupRI   
## GroupWI   
## GroupCONT:Week\_c   
## GroupRI:Week\_c   
## GroupWI:Week\_c   
## GroupCONT:Week\_c\_sq   
## GroupRI:Week\_c\_sq 0.000   
## GroupWI:Week\_c\_sq 0.000 0.000   
## GroupCONT:Week\_c\_cub 0.000 0.000 0.000   
## GroupRI:Week\_c\_cub 0.000 0.000 0.000 0.000   
## GroupWI:Week\_c\_cub 0.000 0.000 0.000 0.000   
## GrpRI:Wk\_c\_c  
## GroupRI   
## GroupWI   
## GroupCONT:Week\_c   
## GroupRI:Week\_c   
## GroupWI:Week\_c   
## GroupCONT:Week\_c\_sq   
## GroupRI:Week\_c\_sq   
## GroupWI:Week\_c\_sq   
## GroupCONT:Week\_c\_cub   
## GroupRI:Week\_c\_cub   
## GroupWI:Week\_c\_cub 0.000   
##   
## Standardized residuals:  
## Min Q1 Med Q3 Max   
## -2.3083393 -0.6242544 -0.0290322 0.6209849 2.8049798   
##   
## Residual standard error: 3.280089   
## Degrees of freedom: 399 total; 387 residual

anova(fit.ar1\_poly\_nested\_centered)

## Denom. DF: 387   
## numDF F-value p-value  
## Group 3 13222.450 <.0001  
## Group:Week\_c 3 7.390 0.0001  
## Group:Week\_c\_sq 3 2.978 0.0315  
## Group:Week\_c\_cub 3 0.924 0.4293

1. Compare the test statistic using orthogonal polynomial contrasts for quadratic trend in 3(b) to the test statistics for quadratic trend in the previous question, 4(d). Are the test statistics the same? Why would the test statistics be different?

# 3(b) results  
trend\_results <- data.frame(  
 Group = c("RI", "WI"),  
 Trend = c("Quadratic", "Quadratic"),  
 Estimate = c(quadratic\_trend\_RI, quadratic\_trend\_WI),  
 `Standard Error` = c(quadratic\_trend\_RI\_se, quadratic\_trend\_WI\_se),  
 `t-value` = c(t\_value\_quadratic\_RI, t\_value\_quadratic\_WI),  
 `p-value` = c(p\_value\_quadratic\_RI, p\_value\_quadratic\_WI)  
)  
print(trend\_results)

## Group Trend Estimate Standard.Error t.value p.value  
## 1 RI Quadratic -0.6682923 0.2912402 -2.294643 0.02230101  
## 2 WI Quadratic -0.3221312 0.2542153 -1.267159 0.20587862

# Create comparison table for 3(b) and 4(d) centered  
comparison\_results <- data.frame(  
 Group = c("RI", "RI", "WI", "WI"),  
 Trend = c("Quadratic", "Quadratic", "Quadratic", "Quadratic"),  
 Question = c("3(b)", "4(d)\_centered", "3(b)", "4(d)\_centered"),  
 Estimate = c(quadratic\_trend\_RI, -0.07766, quadratic\_trend\_WI, -0.03063),  
 `Standard Error` = c(quadratic\_trend\_RI\_se, 0.0305480, quadratic\_trend\_WI\_se, 0.0266645),  
 `t-value` = c(t\_value\_quadratic\_RI, -2.54239, t\_value\_quadratic\_WI, -1.14854),  
 `p-value` = c(p\_value\_quadratic\_RI, 0.0114, p\_value\_quadratic\_WI, 0.2515)  
)  
  
# Display the table  
print(comparison\_results)

## Group Trend Question Estimate Standard.Error t.value p.value  
## 1 RI Quadratic 3(b) -0.6682923 0.2912402 -2.294643 0.02230101  
## 2 RI Quadratic 4(d)\_centered -0.0776600 0.0305480 -2.542390 0.01140000  
## 3 WI Quadratic 3(b) -0.3221312 0.2542153 -1.267159 0.20587862  
## 4 WI Quadratic 4(d)\_centered -0.0306300 0.0266645 -1.148540 0.25150000

## **Graphs**

**poly\_nested\_Centered**

library(ggplot2)  
  
strength\_long1$predicted\_strength\_c <- predict(fit.ar1\_poly\_nested\_centered)  
  
# Plot the linear trend for RI and WI  
ggplot(strength\_long1, aes(x = Week\_c, y = predicted\_strength\_c, color = Group)) +  
 geom\_line(size = 1.5) +  
 labs(title = "Trend for RI and WI: AR1\_poly\_nested\_centered", x = "Centered Week", y = "Predicted Strength") +  
 theme\_minimal()

## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.  
## ℹ Please use `linewidth` instead.  
## This warning is displayed once every 8 hours.  
## Call `lifecycle::last\_lifecycle\_warnings()` to see where this warning was  
## generated.

A graph of a number of people

Description automatically generated with medium confidence Poly

strength\_long1$predicted\_strength\_p <- predict(fit.ar1\_poly)  
  
# Plot the linear trend for RI and WI  
ggplot(strength\_long1, aes(x = Week\_num, y = predicted\_strength\_p, color = Group)) +  
 geom\_line(size = 1.5) +  
 labs(title = "Trend for RI and WI Groups: AR1\_poly", x = "Week", y = "Predicted Strength") +  
 theme\_minimal()

A graph with a line graph

Description automatically generated with medium confidence

Original data trends: individual

library(tidyverse)

## ── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──  
## ✔ dplyr 1.1.4 ✔ readr 2.1.5  
## ✔ forcats 1.0.0 ✔ stringr 1.5.1  
## ✔ lubridate 1.9.3 ✔ tibble 3.2.1  
## ✔ purrr 1.0.2 ✔ tidyr 1.3.1  
## ── Conflicts ────────────────────────────────────────── tidyverse\_conflicts() ──  
## ✖ dplyr::collapse() masks nlme::collapse()  
## ✖ dplyr::filter() masks stats::filter()  
## ✖ dplyr::lag() masks stats::lag()  
## ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors

# Individual plots for each subject  
ggplot(strength\_long1, aes(x = Week, y = Strength, group = ID)) +  
 geom\_line() +  
 facet\_wrap(~ Group) +  
 labs(title = "Individual Trends per Group", x = "Week", y = "Strength")

A graph of different trends

Description automatically generated with medium confidence Original data trends: average

# Plot the average trends for each group  
group\_trend <- strength\_long1 %>%  
 group\_by(Group, Week) %>%  
 summarize(Avg\_Strength = mean(Strength))

## `summarise()` has grouped output by 'Group'. You can override using the  
## `.groups` argument.

ggplot(group\_trend, aes(x = Week, y = Avg\_Strength, color = Group, group = Group)) +  
 geom\_line(size = 1.5) +  
 labs(title = "Average Trend per Group", x = "Week", y = "Average Strength")

A graph with a line graph

Description automatically generated with medium confidence

1. model: fit.ar1\_poly\_nested

# Full model  
fit\_full <- gls(Strength ~ -1 + Group + Week\_num:Group + Week\_sq:Group + Week\_cub:Group,  
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "ML")  
  
# Reduced models (dropping higher-order terms one by one)  
fit\_no\_cub <- gls(Strength ~ -1 + Group + Week\_num:Group + Week\_sq:Group,  
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "ML")  
  
fit\_no\_sq <- gls(Strength ~ -1 + Group + Week\_num:Group,  
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "ML")  
  
fit\_no\_week <- gls(Strength ~ -1 + Group,  
 data = strength\_long1,   
 correlation = corAR1(form = ~ 1 | IDf),   
 method = "ML")  
  
# Perform ANOVA comparisons to get sequential sum of squares  
anova(fit\_no\_cub, fit\_full) # not significant --> drop cub

## Model df AIC BIC logLik Test L.Ratio p-value  
## fit\_no\_cub 1 11 1281.437 1325.315 -629.7183   
## fit\_full 2 14 1284.606 1340.451 -628.3030 1 vs 2 2.83061 0.4185

anova(fit\_no\_sq, fit\_no\_cub) # significant --> keep sq

## Model df AIC BIC logLik Test L.Ratio p-value  
## fit\_no\_sq 1 8 1284.403 1316.315 -634.2017   
## fit\_no\_cub 2 11 1281.437 1325.315 -629.7183 1 vs 2 8.96679 0.0297

anova(fit\_no\_week, fit\_no\_sq) # highly significant --> keep linear

## Model df AIC BIC logLik Test L.Ratio p-value  
## fit\_no\_week 1 5 1299.936 1319.881 -644.9680   
## fit\_no\_sq 2 8 1284.403 1316.315 -634.2017 1 vs 2 21.53253 1e-04